Inhomogeneous structure:
Due to the fields within two guided-wave media, the microstrip does not support a pure TEM wave.

When the longitudinal components of the fields for the dominant mode of a microstrip line is much smaller than the transverse components, the quasi-TEM approximation is applicable to facilitate design.
Microstrip Lines
- Transmission Line Parameters

Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance ($Z_c$)

- For thin conductors (i.e., $t \to 0$), closed-form expression (error $\leq 1\%$):
  - $W/h \leq 1$:
    \[
    \varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{12.575}{W/h} \right)^{0.5} + 0.0025 \left( 1 - \frac{W}{h} \right)
    \]
  - $W/h \geq 1$:
    \[
    \varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{12.575}{W} \right)^{0.5}
    \]

- For thin conductors (i.e., $t \to 0$), more accurate expressions:
  - Effective dielectric constant (error $\leq 0.2\%$):
    \[
    \varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{10}{u} \right)^{0.5}
    \]
    \[
    a = 1 + \frac{1}{40} \ln \left( \frac{u^2 + \frac{\varepsilon_r - 1}{2}}{u^2 + 0.432} \right) + \frac{1}{18.7} \ln \left( 1 + \frac{u}{18.1} \right)
    \]
    \[
    b = 0.56 \left( \frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right)^{0.05}
    \]

  - Characteristic impedance (error $\leq 0.03\%$):
    \[
    Z_c = \frac{\eta}{\sqrt{\varepsilon_{re}}} \left( \frac{W}{h} \right)^{0.25} \left( \frac{W}{h} \right) + 1.393 + 0.677 \ln \left( \frac{W}{h} + 1.444 \right)
    \]

Guided wavelength
\[
\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}}
\]
or
\[
\lambda_g = \frac{300}{f(GHz)\sqrt{\varepsilon_{re}}} mm
\]

Propagation constant
\[
\beta = \frac{2\pi}{\lambda_g}
\]

Phase velocity
\[
\nu_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon_{re}}}
\]

Electrical length
\[
\theta = \beta \ell
\]

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Microstrip Lines
- Transmission Line Parameters

- Losses
  - Conductor loss
  - Dielectric loss
  - Radiation loss

- Dispersion
  - $\varepsilon_r(f)$
  - $Z_0(f)$

- Surface Waves and higher-order modes
  - Coupling between the quasi-TEM mode and surface wave mode become significant when the frequency is above $f_s$
    
    $$f_s = \frac{c \tan^{-1} \varepsilon_r}{\sqrt{2\pi h \sqrt{\varepsilon_r - 1}}}$$

  - Cutoff frequency $f_c$ of first higher-order modes in a microstrip
    
    $$f_c = \frac{c}{\sqrt{\varepsilon_r (2W + 0.8h)}}$$

  - The operating frequency of a microstrip line $< \text{Min}(f_s, f_c)$

Microstrip Lines
- Tx-Line

- Synthesis of transmission line – electrical or physical parameters

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The coupled line structure supports two quasi-TEM modes: odd mode and even mode.

- **Odd mode**
  - Electric wall
  - Magnetic wall
  - Electric field
  - Magnetic field

- **Even mode**
  - Electric wall
  - Magnetic wall
  - Electric field
  - Magnetic field

Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance ($Z_c$)

- **Odd mode**
  - Characteristic impedance ($Z_{co}$)
  - Effective dielectric constant ($\varepsilon_{re}^o$)

- **Even mode**
  - Characteristic impedance ($Z_{ce}$)
  - Effective dielectric constant ($\varepsilon_{re}^e$)

- **Odd- and Even- Mode**: The characteristic impedances ($Z_{co}$ and $Z_{ce}$) and effective dielectric constants ($\varepsilon_{re}^o$ and $\varepsilon_{re}^e$) are obtained from the capacitances ($C_o$ and $C_e$):
  - **Odd-Mode**:
    - $Z_{co} = (c\sqrt{C_o^a C_o})^{-1}$
    - $\varepsilon_{re}^o = C_o^a / C_o$
  - **Even-Mode**:
    - $Z_{ce} = (c\sqrt{C_e^a C_e})^{-1}$
    - $\varepsilon_{re}^e = C_e^a / C_e$

- $C_o^a$ and $C_e^a$ are even- and odd-mode capacitances for the coupled microstrip line configuration with air as dielectric.
**Coupled Lines**

− **Odd- and Even- Mode**

Effective Dielectric Constant ($\varepsilon_{re}$) and Characteristic Impedance ($Z_C$)

- Odd- and Even- Mode Capacitances:
  - **Odd-Mode:**
    \[ C_o = C_p + C_f + C_{gd} + C_{ga} \]
  - **Even-Mode:**
    \[ C_e = C_p + C_f + C_f' \]

- $C_p$ denotes the parallel plate capacitance between the strip and the ground plane:
  \[ C_p = \varepsilon_o \varepsilon_r \frac{W}{h} \]

- $C_f$ is the fringe capacitance as if for an uncoupled single microstrip line:
  \[ 2C_f = \sqrt{\varepsilon_{re}(cZ_0)} - C_p \]

- $C_f'$ accounts for the modification of fringe capacitance $C_f$:
  \[ C_f' = \frac{C_f}{1 + A(h/s)\tanh(8s/h)} \quad A = \exp[-0.1 \exp(2.33 - 2.53W/h)] \]

- $C_{gd}$ may be found from the corresponding coupled stripline geometry:
  \[ C_{gd} = \frac{\varepsilon_o \varepsilon_r}{\pi} \ln\left[\coth\left(\frac{\pi}{4}\frac{s}{h}\right)\right] + 0.65C_f \left(\frac{0.02\sqrt{\varepsilon_r}}{s/h} + 1 - \frac{1}{\varepsilon_r^2}\right) \]

- $C_{ga}$ can be modified from the capacitance of the corresponding coplanar strips:
  \[ C_{ga} = \varepsilon_o \frac{K(k')}{K(k)} \]

**Discontinuities And Components**

− **Discontinuities**

Microstrip discontinuities commonly encountered in the layout of practical filters include steps, open-ends, bends, gaps, and junctions.

The effects of discontinuities can be accurately modeled by full-wave EM simulator or closed-form expressions and taken into account in the filter designs.

- **Steps in width:**

- **Open ends:**

- **Gaps:**

- **Bends:**
Discontinuities
– Steps in width

\[ C = 0.00137h \frac{\sqrt{\varepsilon_{\text{re}}} \varepsilon_{\text{rel}}}{Z_{c1}} \left(1 - \frac{W_2}{W_1}\right) \left(\frac{\varepsilon_{\text{rel}} + 0.3}{\varepsilon_{\text{rel}} - 0.258}\right) \left(\frac{W_1/h + 0.264}{W_1/h + 0.8}\right) \text{(pF)} \]

\[ L_1 = \frac{L_{w1}}{L_{w1} + L_{w2}} L \quad L_{w2} = \frac{L_{w2}}{L_{w1} + L_{w2}} L \]

where

\[ L = 0.000987h \left(1 - \frac{Z_{c1}}{Z_{c2}} \sqrt{\frac{\varepsilon_{\text{rel}}}{\varepsilon_{\text{re}}}}\right)^2 \]

Note: \( L_{w1} \) for \( i = 1, 2 \) are the inductances per unit length of the appropriate microstrips, having widths \( W_1 \) and \( W_2 \), respectively.

\( Z_{c1} \) and \( \varepsilon_{\text{rel}} \) denote the characteristic impedance and effective dielectric constant corresponding to width \( W_i \), and \( h \) is the substrate thickness in micrometers.

Discontinuities
– Open ends

The fields do not stop abruptly but extend slightly further due to the effect of the fringing field.

\[ \Delta l = \frac{cZ_cC_p}{\sqrt{\varepsilon_{\text{re}}}} \]

Closed-form expression:

\[ \Delta l = \frac{\xi_1 \xi_2 \xi_3}{\xi_4} \]

where

\[ \xi_1 = 0.434907 \frac{\varepsilon_{\text{re}}^{0.81} + 0.26(W/h)^{0.8544} + 0.236}{\varepsilon_{\text{rel}}^{0.81} - 0.189(W/h)^{0.8544} + 0.87} \]

\[ \xi_2 = 1 + \frac{(W/h)^{0.371}}{2.35\varepsilon_r + 1} \]

\[ \xi_3 = 1 + \frac{0.5274 \tan^{-1}[0.084(W/h)^{1.9413\xi_2}]}{\varepsilon_{\text{re}}^{0.9236}} \]

\[ \xi_4 = 1 + 0.037 \tan^{-1}[0.067(W/h)^{1.456}] \{6 - 5 \exp[0.036(1 - \varepsilon_r)]\} \]

\[ \xi_5 = 1 - 0.218 \exp(-7.5W/h) \]

The accuracy is better than 0.2 % for the range of 0.01 \( \leq W/h \leq 100 \) and \( \varepsilon_r \leq 128 \)

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Discontinuities
- Gaps

\[ C_o \left( \frac{\text{pF/m}}{W} \right) = \left( \frac{\varepsilon_r}{9.6} \right)^{0.8} \left( \frac{s}{W} \right)^{0.9} \exp(k_o) \]
\[ C_e \left( \frac{\text{pF/m}}{W} \right) = 12 \left( \frac{\varepsilon_r}{9.6} \right)^{0.9} \left( \frac{s}{W} \right)^{0.8} \exp(k_e) \]

\[ C_p = 0.5C_e \]
\[ C_g = 0.5C_o - 0.25C_e \]

\[ m_s = \frac{W}{h} [0.619 \log(W/h) - 0.3853] \]
\[ k_o = 4.26 - 1.453 \log(W/h) \]
\[ m_e = 0.8675 \]
\[ k_e = 2.043 \left( \frac{W}{h} \right)^{0.12} \]
\[ m_e = \frac{1.565}{(W/h)^{0.16}} - 1 \]
\[ k_e = 1.97 - \frac{0.03}{W/h} \]

The accuracy is within 7% for 0.5 ≤ W/h ≤ 2 and 2.5 ≤ ε_r ≤ 15

- Bends

\[ C \left( \frac{\text{pF/m}}{W} \right) = \begin{cases} 
(14\varepsilon_r + 12.5)W/h - (1.83\varepsilon_r - 2.25) & \text{for } W/h < 1 \\
(9.5\varepsilon_r + 1.25)W/h + 5.2\varepsilon_r + 7.0 & \text{for } W/h \geq 1 
\end{cases} \]

\[ \frac{L}{h} \left( \frac{\text{nH/m}}{W} \right) = 100 \left( 4 \sqrt{\frac{W}{h}} - 4.21 \right) \]

The accuracy on the capacitance is quoted as within 5% over the ranges of 2.5 ≤ ε_r ≤ 15 and 0.1 ≤ W/h ≤ 5.
The accuracy on the inductance is about 3% for 0.5 ≤ W/h ≤ 2.
Components
– lumped inductors and capacitors

- Lumped inductors and capacitors
  The elements whose physical dimensions are much smaller than the free space wavelength \( \lambda_0 \) of the highest operating frequency (smaller than 0.1 \( \lambda_0 \)).

- Design of inductors
  - High-impedance line
  - Meander line
  - Circular spiral
  - Square spiral

- Circuit representation

- Initial design formula for straight-line inductor
  \[
  L(\text{nH}) = 2 \times 10^{-4} \left[ \ln \left( \frac{W + t}{I} \right) + 1.193 + 0.2235 \frac{W + t}{I} \right] K_g \quad \text{for } l \text{ in } \mu\text{m}
  
  R = \frac{R_f}{2(W + t)} \left[ 1.4 + 0.217 \ln \left( \frac{W}{St} \right) \right] \quad \text{for } 5 < \frac{W}{l} < 100
  \]

Components
– lumped inductors and capacitors

- Design of capacitors
  - Interdigital capacitor
    Assuming the finger width \( W \) equals to the space and empirical formula for capacitance is shown as follow

  \[
  C(\text{pF}) = 3.937 \times 10^{-5} R(t_e + 1)[0.11(n - 3) + 0.252] \quad \text{for } l \text{ in } \mu\text{m}
  
  R = \frac{4 R_f}{3 W \pi}
  \]

  - Metal-insulator-metal (MIM) capacitor
    Estimation of capacitance and resistance is approximated by parallel-plate

    \[
    C = \frac{\varepsilon(W \times l)}{d}
    
    R = \frac{R_f}{W}
    \]

- Circuit representation
Components

– Quasilumped elements (1)

- Quasilumped elements
  - Physical lengths are smaller than a quarter of guided wavelength $\lambda_g$.
    - High-impedance short line element

  \[
  \begin{bmatrix}
  A & B \\
  C & D \\
  \end{bmatrix} = \begin{bmatrix}
  \cos \beta t & jZ \sin \beta t \\
  -jZ \sin \beta t & \cos \beta t \\
  \end{bmatrix}
  \]

- Derivation

  \[
  \begin{align*}
  Z_{ii} &= \frac{1}{Z_c} \frac{1}{jX} \\
  Y_{ii} &= \frac{\cos \beta t}{jZ_c} - \frac{1}{jZ_c} \\
  \end{align*}
  \]

Components

– Quasilumped elements (2)

- Quasilumped elements
  - Low-impedance short line element

  \[
  \begin{bmatrix}
  A & B \\
  C & D \\
  \end{bmatrix} = \begin{bmatrix}
  \cos \beta t & jZ \sin \beta t \\
  -jZ \sin \beta t & \cos \beta t \\
  \end{bmatrix}
  \]

- Derivation

  \[
  \begin{align*}
  Z_{ii} &= \frac{Z_c}{j \sin \beta t} \\
  B &= \sin \beta t / Z_c \\
  \end{align*}
  \]
Components

– Quasilumped elements (3)

- Quasilumped elements
  - Open- and short-circuited stubs (assuming the length \( L \) is smaller than a quarter of guided wavelength \( \lambda_g \))

\[
Y_m = j Y_L \tan \left( \frac{2\pi L}{\lambda_g} \right) = j Y_L \left( \frac{2\pi L}{\lambda_g} \right) = j Y_L \left( \frac{2\pi L}{\lambda_g} \right)
\]

\[
Z_m = j Z_L \tan \left( \frac{2\pi L}{\lambda_g} \right) = j Z_L \left( \frac{2\pi L}{\lambda_g} \right) = j Z_L \left( \frac{2\pi L}{\lambda_g} \right)
\]

\( l < \frac{\lambda_g}{8} \)

Components

– Resonators

- Lumped-element or quasilumped-element resonators
  - Formed by the lumped or quasilumped inductors and capacitors
  - Resonate at \( \omega_0 = \frac{1}{\sqrt{LC}} \)

- Distributed resonators
  - Quarter-wavelength resonators, half-wavelength resonators, and patch resonators.
Loss Considerations for Microstrip Resonators

- Unloaded quality factor $Q_u$ is served as a justification for whether or not the required insertion loss of a bandpass filter can be met.

$$Q_u = \frac{\omega}{\omega_p}$$

- Time-average energy stored in resonator
- Average power lost in resonator

- The total unloaded quality factor of a resonator can be found by adding conductor, dielectric, and radiation loss together.

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r}$$

- Quality factors $Q_c$ and $Q_d$ for a microstrip line

$$Q_c = \frac{\pi}{\alpha_c \lambda_g}$$

$$Q_d \geq \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan \delta}$$

or

$$Q_d = \frac{\pi}{\alpha_d \lambda_g}$$

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