Transmission Line and Microwave Networks

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2.3 Microwave network analysis

- Impedance and admittance matrices

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ + I_n^-,$$

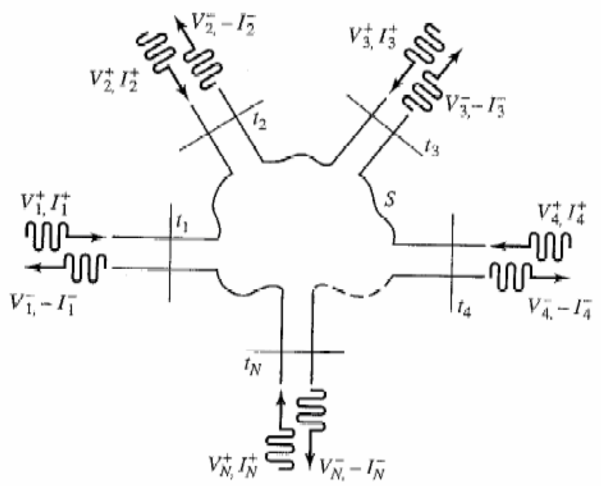

FIGURE 2.12 An arbitrary N-port microwave network.

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdot & \cdot & \cdot & Z_{1N} \\ Z_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{N1} & \cdot & \cdot & \cdot & \cdot & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix},$$

$$[V] = [Z][I].$$

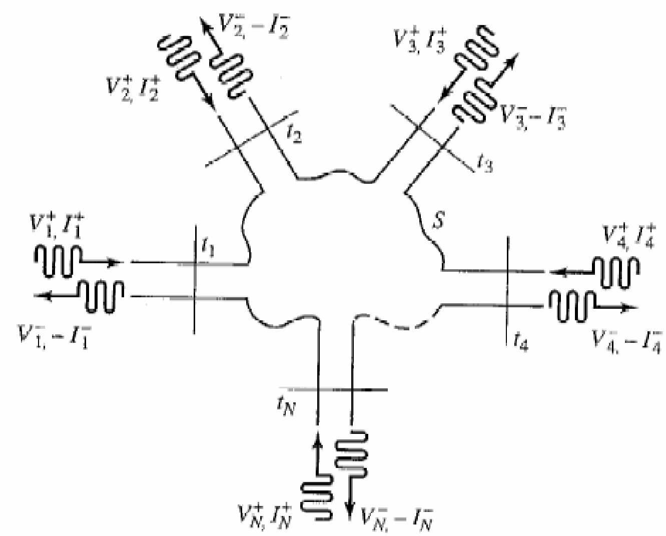

FIGURE 2.12 An arbitrary N-port microwave network.

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot & \cdot & Y_{1N} \\ Y_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{N1} & \cdot & \cdot & \cdot & \cdot & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_N \end{bmatrix},$$

$$[I] = [Y][V].$$

- Short-circuit admittance matrix

$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k = 0 \text{ for } k \neq j},$$

- Elements in Z and Y are complex in general.
- $Z_{ij}=Z_{ji}$ and $Y_{ij}=Y_{ji}$ if network is reciprocal.

◆ Zij and Yij are pure imaginary for a lossless network

$$P_{avg} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}([Z][I])^t[I]^* = \frac{1}{2}[I]^t[Z][I]^*$$
$$= \frac{1}{2}(I_1 Z_{11} I_1^* + I_1 Z_{12} I_2^* + I_2 Z_{21} I_1^* + \cdots)$$
$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} I_m Z_{mn} I_n^*$$

$$\mathrm{Re}\{I_m Z_{mm} I_m^*\} = |I_m|^2 \mathrm{Re}\{Z_{mm}\} = 0,$$

$$\mathrm{Re}\{Z_{mm}\} = 0.$$

Next, let all port currents be zero except for I_m and I_n. Then (2.57) reduces to

$$\mathrm{Re}\{(I_n I_m^* + I_m I_n^*) Z_{mn}\} = 0,$$

since $Z_{mn} = Z_{nm}$ for a reciprocal network. But $(I_n I_m^* + I_m I_n^*)$ is a purely real quantity which is, in general, nonzero. Thus we must have that

$$\mathrm{Re}\{Z_{mn}\} = 0. \qquad (2.59)$$

Scattering matrix

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ \vdots & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix},$$

$$[V^-] = [S][V^+].$$

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}.$$

- Two-port case

◆ The relation between Z and S

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ + I_n^- = V_n^+ - V_n^-.$$

$$[Z][I] = [Z][V^+] - [Z][V^-] = [V] = [V^+] + [V^-],$$

$$([Z] + [U])[V^-] = ([Z] - [U])[V^+],$$

$$[U] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & & & & \cdot \\ \cdot & & \cdot & & & \cdot \\ \cdot & & & \cdot & & \cdot \\ \cdot & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}.$$

$$[S] = ([Z] + [U])^{-1}([Z] - [U]),$$

EXAMPLE 2.4 EVALUATION OF SCATTERING PARAMETERS

Find the S parameters of the matched 3 dB attenuator circuit shown in Figure 2.13.

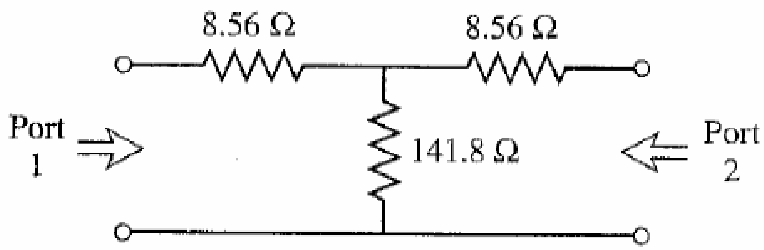

FIGURE 2.13 A matched 3 dB attenuator with a 50 Ω characteristic impedance.

Solution

From (2.61), S_{11} can be found as the reflection coefficient seen at port 1 when port 2 is terminated in a matched load ($Z_0 = 50\ \Omega$):

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma^{(1)}\Big|_{V_2^+=0} = \left.\frac{Z_{\text{in}}^{(1)} - Z_0}{Z_{\text{in}}^{(1)} - Z_0}\right|_{Z_0 \text{ on port 2}},$$

where $Z_{\text{in}}^{(1)}$ is the input impedance seen at port 1 when port 2 is terminated with a matched load. With reference to Figure 2.13, this can be calculated as

$$Z_{\text{in}}^{(1)} = \frac{8.56 + [141.8(8.56 + 50)]}{(141.8 + 8.56 + 50)} = 50\ \Omega,$$

S_{21} can be found by applying an incident wave at port 1, V_1^+, and measuring the outcoming wave at port 2, V_2^-. This is equivalent to the transmission coefficient from port 1 to port 2:

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}.$$

From the fact that $S_{11} = S_{22} = 0$, we know that $V_1^- = 0$ when port 2 is terminated in $Z_0 = 50\ \Omega$, and that $V_2^+ = 0$. In this case we then have that $V_1^+ = V_1$ and $V_2^- = V_2$. So by applying a voltage V_1 at port 1 and using voltage division twice we can find the voltage across the 50 Ω load resistor at port 2:

$$V_2^- = V_2 = V_1\left(\frac{41.44}{41.44+8.56}\right)\left(\frac{50}{50+8.56}\right) = 0.707V_1,$$

where 41.44 Ω is the resistance resulting from the parallel combination of the 50 Ω load and the 8.56 Ω and 141.8 Ω resistors in series. Thus, $S_{12} = S_{21} = 0.707$.

If the input power is $|V_1^+|^2/2Z_0$, then the output power is

$$\frac{|V_2^-|^2}{2Z_0} = \frac{|S_{21}V_1^+|^2}{2Z_0} = \frac{|S_{21}|^2|V_1^+|^2}{2Z_0} = \frac{|V_1^+|^2}{4Z_0},$$

which is one-half of the input power, as expected for a 3 dB attenuator.

◆Transmission matrix (ABCD) matrix

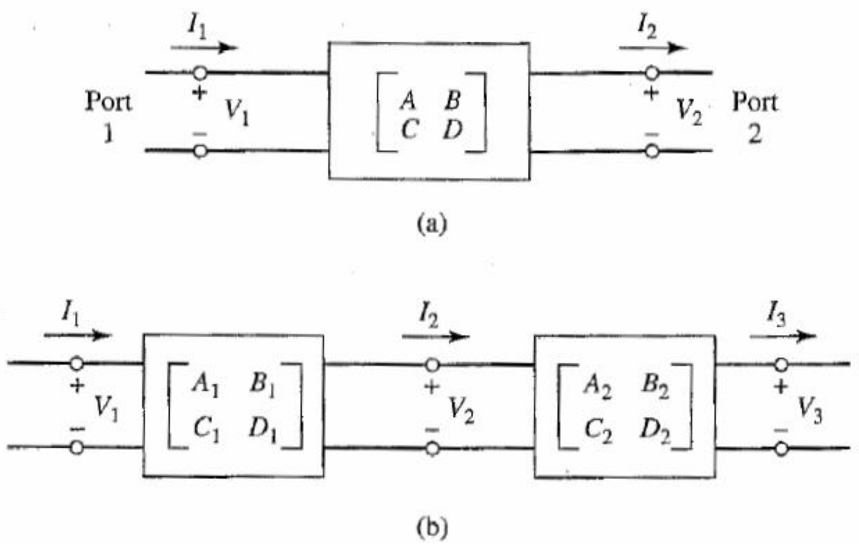

FIGURE 2.14 (a) A two-port network; (b) a cascade connection of two-port networks.

$$V_1 = AV_2 + BI_2,$$
$$I_1 = CV_2 + DI_2,$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}.$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix},$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}.$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix},$$

Conversion of ABCD to Z

$$V_1 = I_1 Z_{11} - I_2 Z_{12},$$
$$V_2 = I_1 Z_{21} - I_2 Z_{22}.$$

$$V_1 = A V_2 + B I_2,$$
$$I_1 = C V_2 + D I_2,$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} = \frac{Z_{11}}{Z_{21}},$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \left.\frac{I_1 Z_{11} - I_2 Z_{12}}{I_2}\right|_{V_2=0} = Z_{11} \left.\frac{I_1}{I_2}\right|_{V_2=0} - Z_{12}$$
$$= Z_{11} \frac{Z_{22}}{Z_{21}} - Z_{12} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} = \frac{1}{Z_{21}},$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_2 Z_{22}/Z_{21}}{I_2} = \frac{Z_{22}}{Z_{21}}.$$