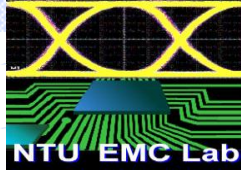


Antenna for EMC

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Introduction

1. Overview
2. Steps in evaluation of radiated fields
3. Ideal Dipole (Hertzian Dipole)
4. Antenna parameters
5. Dipole and Monopole
6. Small Loop Antenna
7. Antenna Arrays
8. Examples of Antenna
9. Antennas in communication systems



How antenna radiate: a single accelerated charged particle

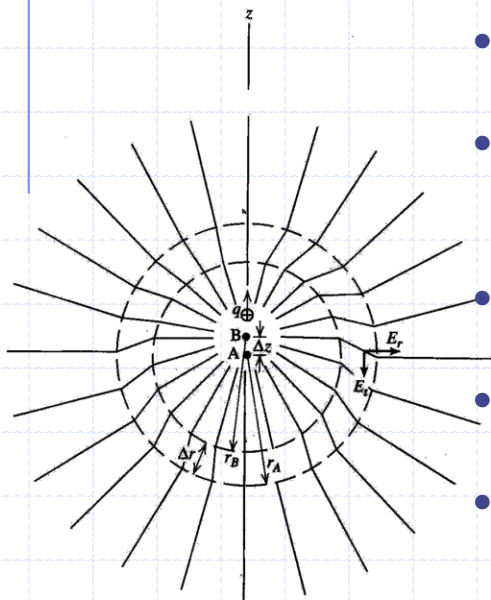


Figure 1-2 Illustration of how an accelerated charged particle radiates. Charge q moves with constant velocity in the $+z$ -direction until it reaches point A (time $t = 0$), after which it accelerates to point B (time $t = \Delta t$) and then maintains its velocity. The electric field lines shown here are for a time r_B/c after the charge passed point B.

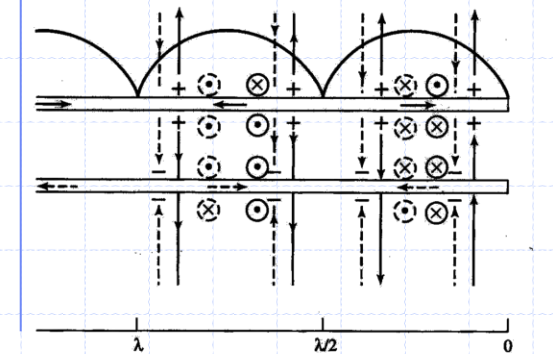
- The static electric field originates at charge and is directed radially away from charge.
- At point A, the charge begins to be accelerated until reaching point B.
- The distance between the circles is that distance light would travel in time Δt , and $\Delta r = r_b - r_a = \Delta t * c$
- Charge moves slowly compared to the speed of the light, $\therefore \Delta r \gg \Delta z$ and two circles are *concentric*.
- The electric field lines in the Δr region are joined together because of required continuity of electrical lines in the absence of charges.
- This disturbance expands outward and has a transverse component E_t , which is the radiated field.
- If charges are accelerated back and forth (i.e., oscillate), a regular disturbance is created and radiation is continuous.
- This disturbance is directly analogous to a transient wave created by a stone dropped into a calm lake.



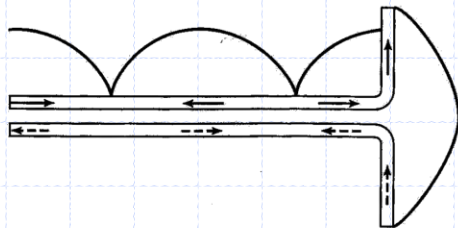
How antenna radiate : Evolution of a dipole antenna from an open-circuited transmission line

• Open-circuited transmission line

1. The currents are in opposite directions on the two wires and behaves as a standing wave pattern with a zero current magnitude at the ends and every half wavelength from the end.
2. The conductors guide the waves and the power resides in the region surrounding the conductors as manifested by the electric and magnetic fields.
3. Electric fields originate from or terminate on charges and perpendicular to the wires.
4. Magnetic fields encircle the wires.



(a) Open-circuited transmission line showing currents, charges, and fields. The electric fields are indicated with lines and the magnetic fields with arrow heads and tails, solid (dashed) for those arising from the top (bottom) wire.



(b) Peak currents on a half-wavelength dipole created by bending out the ends of the transmission line.

Figure 1-3 Evolution of a dipole antenna from an open-circuited transmission line.

• Bending outward to form a dipole

1. The currents are no longer opposite but are both upwardly directed.
2. The bounded fields are exposed to the space.
3. The currents on the dipole are approximately sinusoidal.
4. The situation on the Fig. is the peak current condition. As time proceeds and current oscillation occur, the disturbed fields are radiated.



How antenna radiate : Time dynamics of the fields for a dipole antenna

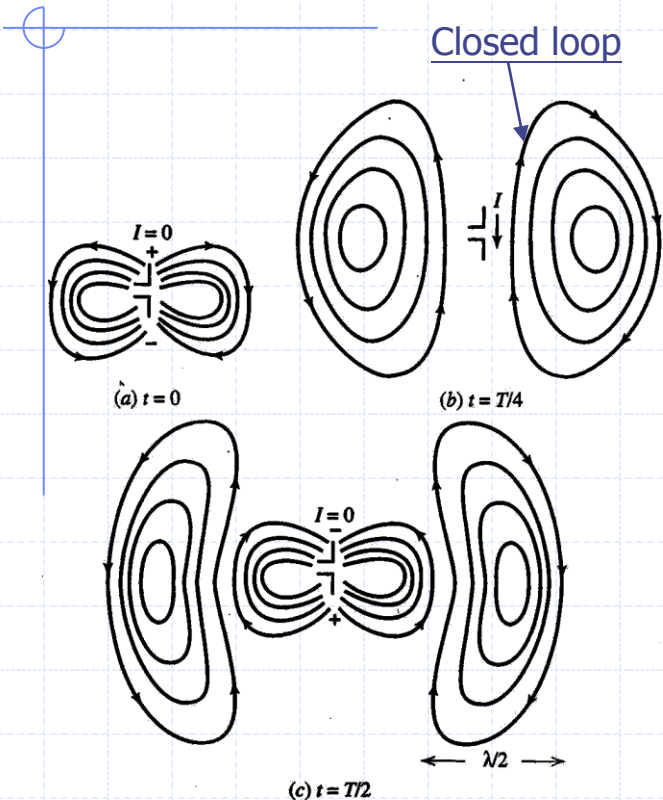


Figure 1-4 Electric fields of an oscillating dipole for various instants of time. The oscillations are of frequency f with a period of $T = 1/f$.

- At $t = 0$, peak charges buildup occurs (positive on the upper half and negative on the lower half). current $I = 0$.
- At $t = T/4$, $+/-$ charges are neutralized and I is maximum.
- Because there are no longer charges for the termination of electric fields, they form **closed loop** near the dipole.
- At $t = T/2$, peak charges buildup again, but upper half is negative and lower half is positive. $I = 0$.
- Notice the definition of $\lambda/2$ at $t = T/2$.



Overview of antenna

- **Electrically small antennas:** The extent of the antenna structure is much less than a wavelength λ .

Properties:

- Very low directivity
- Low input resistance
- High input reactance
- Low radiation efficiency

Examples:



Short dipole



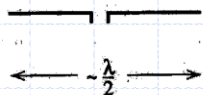
Small loop

- **Resonant antennas:** The antenna operates well at a single or selected narrow frequency bands.

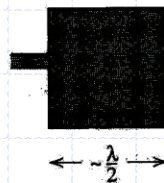
Properties:

- Low to moderate gain
- Real input impedance
- Narrow bandwidth

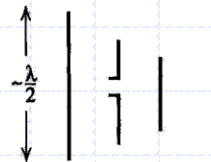
Examples:



Half-wave dipole



Microstrip patch



Yagi



Overview of antenna

- **Broadband antennas:** The pattern, gain, and impedance remain acceptable and are nearly constant over a wide frequency range, and are characterized by an active region with a circumference of one wavelength or an extent of a half-wavelength, which relocates on the antenna as frequency changes.

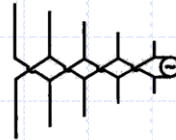
Properties:

- Low to moderate gain
- Constant gain
- Real input impedance
- Wide bandwidth

Examples:



Spiral



Log periodic dipole array

Figure 1-6 Types of antennas.

- **Aperture antennas:** Has a physical aperture (opening) through which waves flow.

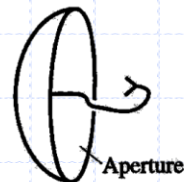
Properties:

- High gain
- Gain increases with frequency
- Moderate bandwidth

Examples:



Horn

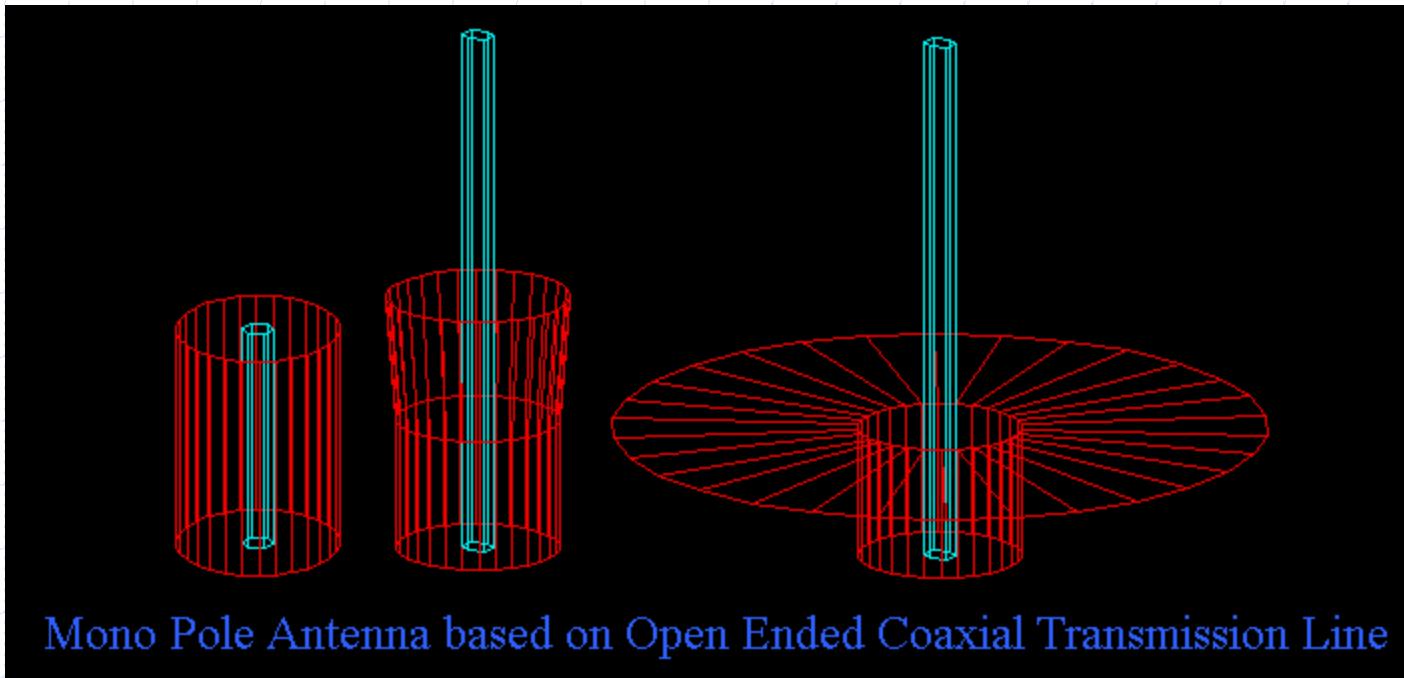


Reflector

Figure 1-6 (continued).

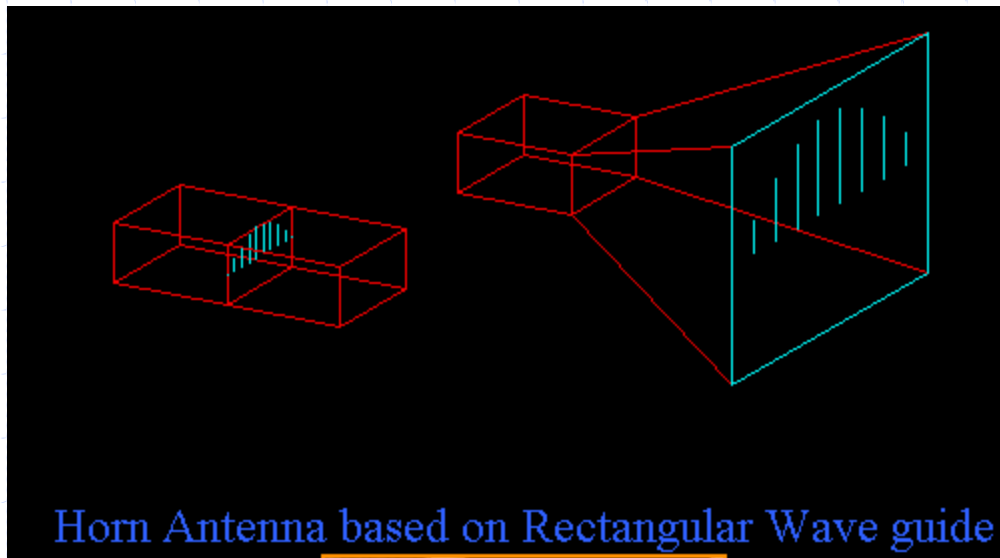


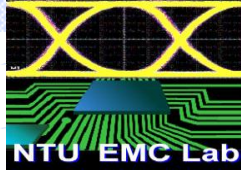
Antenna Concept (I)





Antenna Concept (II)





Steps in evaluation of radiation fields : Solution of Maxwell equations for radiation problems)

$$\nabla \cdot \vec{H} = 0$$

$$\rightarrow \vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

Magnetic vector potential

Electric scalar potential

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\rightarrow \nabla \times (\vec{E} + j\omega\vec{A}) = 0$$

$$\rightarrow \vec{E} + j\omega\vec{A} = -\nabla\phi$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \vec{J}$$

$$\rightarrow \nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = j\omega\mu\epsilon(-j\omega\vec{A} - \nabla\phi) + \mu\vec{J}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

If we set $\nabla \cdot \vec{A} = -j\omega\mu\epsilon\phi$

(Lorenz condition)

Vector wave equation

$$\rightarrow \nabla^2 \vec{A} + \omega^2 \mu\epsilon \vec{A} = -\mu\vec{J}$$

Solution is

$$\vec{A} = \iiint_V \mu\vec{J} \frac{e^{-j\beta R}}{4\pi R} dv'$$

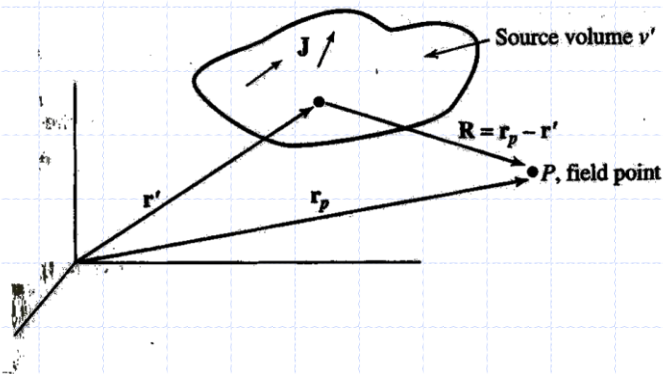


Figure 1-8 Vectors used to solve radiation problems.



Steps in evaluation of radiation fields (Far Fields)

1. Find **A**

Far field

$$\vec{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} \vec{J} e^{j\beta \hat{r} \cdot \mathbf{r}'} dv'$$

Z-directed sources

$$\vec{A} = \hat{z} \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} J_z e^{j\beta \hat{r} \cdot \mathbf{r}'} dv'$$

Z-directed line sources

$$\vec{A} = \hat{z} \mu \frac{e^{-j\beta r}}{4\pi r} \iiint_{v'} I(z') e^{j\beta z' \cdot \cos\theta} dv'$$

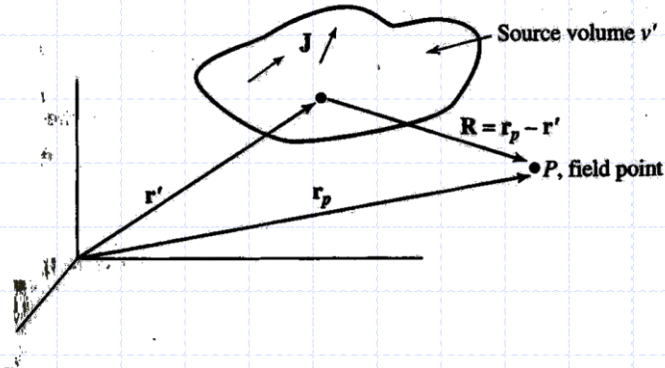
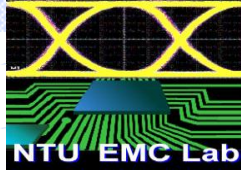


Figure 1-8 Vectors used to solve radiation problems.



Steps in evaluation of radiation fields (Far Fields)

2. Find **E**

Far field

$$\vec{E} = -j\omega \vec{A}$$

Transverse to the propagation direction **r**

$$\vec{E} = -j\omega (A_\theta \hat{\theta} + A_\phi \hat{\phi})$$

Z-directed sources

$$\vec{E} = j\omega \sin \theta A_z \hat{\theta}$$

3. Find **H**

Far field (Plane wave)

$$\vec{H} = \frac{1}{\eta} \hat{r} \times \vec{E}$$

Z-directed source

$$H_\phi = \frac{1}{\eta} E_\theta$$



Far-Field Conditions

Parallel ray approximation for far-field calculations

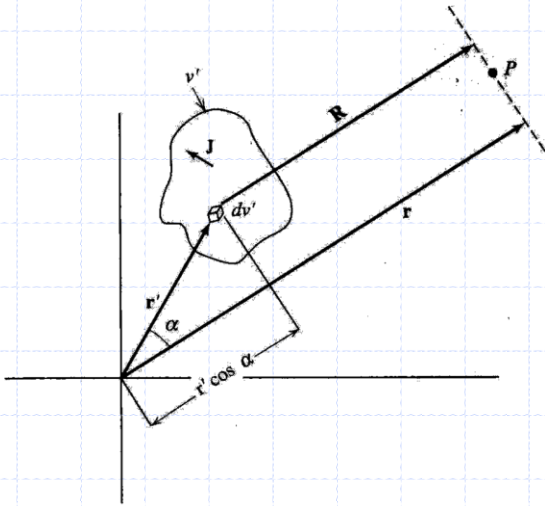


Figure 1-13 Parallel ray approximation for far-field calculations of a general source.

$$R = r - \hat{r} \cdot \vec{r}'$$

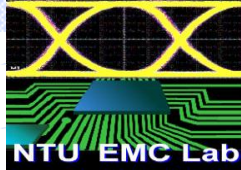
Far Field conditions

$$r > \frac{2D^2}{\lambda}$$

$$r \gg D$$

$$r \gg \lambda$$

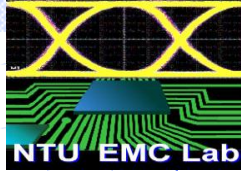
D is the length of line source



Far-Field and Near-field Conditions

天線近場與遠場之特性 R:距離 D:天線尺寸

近場 Near Field ^[1]	遠場 Far Field ^[1]
$R \ll 2D^2/\lambda, D > \lambda$ $R \ll \lambda/6, D < \lambda$	$R \gg 2D^2/\lambda, D > \lambda$ $R \gg \lambda/6, D < \lambda$
Radiation Pattern Depend on Range ^[1]	Radiation Pattern Independent of Range ^[1]
Wave Impedance in air E/H becomes Reactive ^[2]	Wave Impedance in air $E/H = 120 \pi$ is Resistive
High-Impedance Wave ^[2] $E \rightarrow 1/R^3, H \rightarrow 1/R^2$ $R \ll \lambda/6, D < \lambda$ $E/H > 120 \pi$ The E Field ^[3]	Locally Uniform Plane TEM Wave
Low-Impedance Wave ^[2] $E \rightarrow 1/R^2, H \rightarrow 1/R^3$ $R \ll \lambda/6, D < \lambda$ $E/H < 120 \pi$ The H Field ^[3]	



The ideal dipole (Hertzian dipole) : definition

Ideal dipole:

a **uniform amplitude** current that is electrically small with **$\Delta z \ll \lambda$**

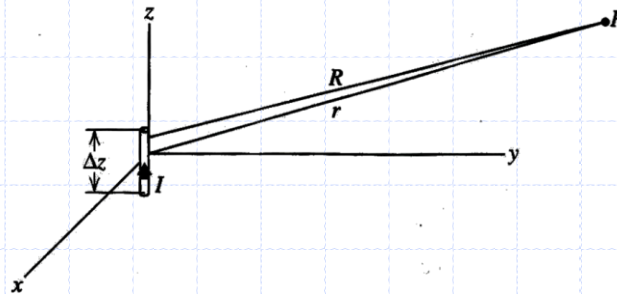


Figure 1-9 The ideal dipole. The current I is uniform, $\Delta z \ll \lambda$, and $R \approx r$.

$$\vec{A} = \hat{z} \mu I \int_{-\Delta z/2}^{\Delta z/2} \frac{e^{-j\beta R}}{4\pi R} dz'$$

$\therefore R \sim r$ for small dipole

$$\vec{A} = \mu I \Delta z \frac{e^{-j\beta r}}{4\pi r} \hat{z}$$



The ideal dipole (Hertzian dipole): \vec{E} and \vec{H} field

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \longrightarrow \vec{H} = \frac{I\Delta z}{4\pi} j\beta \left(1 + \frac{1}{j\beta r}\right) \frac{e^{-j\beta r}}{r} \sin \theta \hat{\phi}$$

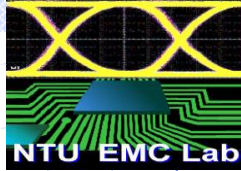
$$\vec{E} = \frac{1}{j\omega\epsilon} \nabla \times \vec{H} \longrightarrow \vec{E} = \frac{I\Delta z}{4\pi} j\omega\mu \left[1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2}\right] \frac{e^{-j\beta r}}{r} \sin \theta \hat{\theta} +$$
$$\frac{I\Delta z}{2\pi} \eta \left[\frac{1}{r} - j\frac{1}{\beta r^2}\right] \frac{e^{-j\beta r}}{r} \cos \theta \hat{r}$$

Far field condition : $\beta r \gg 1$

$$\vec{H} = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-j\beta r}}{r} \sin \theta \hat{\phi}$$

$$\vec{E} = \frac{I\Delta z}{4\pi} j\beta \frac{e^{-j\beta r}}{r} \sin \theta \hat{\theta}$$

$$\frac{E_{\theta}}{H_{\phi}} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} = \eta = 120\pi = 377$$



The ideal dipole (Hertzian dipole): radiated power

Power flowing density : (Unit : w/m*m)

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2} \left(\frac{I \Delta z}{4\pi} \right)^2 \omega \mu \beta \frac{\sin^2 \theta}{r^2} \hat{r}$$

Total radiated power (Unit : W)

$$P = \iint \vec{S} \cdot d\vec{s} = \frac{\omega \mu \beta}{12\pi} (I \Delta z)^2 = 40\pi^2 \left(\frac{\Delta z}{\lambda} \right)^2 I^2$$

Antenna Parameters: radiation pattern

The field pattern with its maximum value is 1 $\longrightarrow F(\theta, \phi) = \frac{E_{\theta}}{E_{\theta, \max}}$

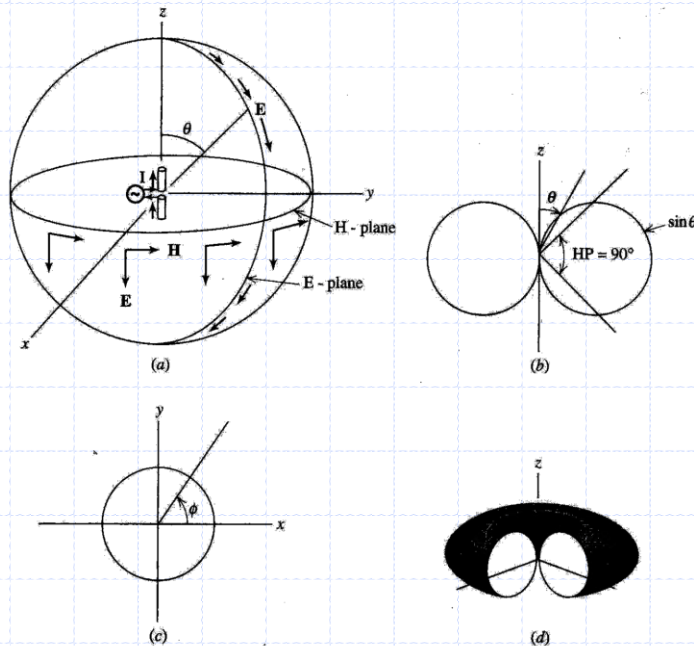


Figure 1-10 Radiation from an ideal dipole. (a) Field components. (b) E-plane radiation pattern polar plot of $|E_{\theta}|$ or $|H_{\phi}|$. (c) H-plane radiation pattern polar plot of $|E_{\theta}|$ or $|H_{\phi}|$. (d) Three-dimensional plot of radiation pattern.

For Hertzian dipole

$$F(\theta) = \sin \theta$$



Antenna Parameters: power pattern

$$P(\theta, \phi) = |F(\theta, \phi)|^2$$

It is worth noting that the field pattern and power pattern are the same in decibels.

$$P(\theta, \phi)_{dB} = |F(\theta, \phi)|_{dB}$$

Power pattern parameters

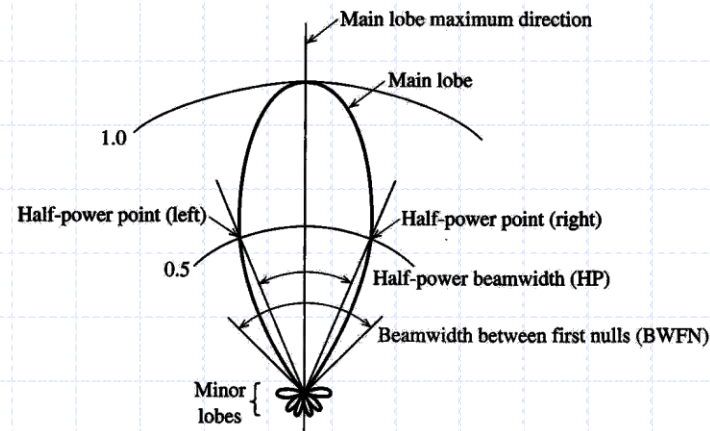


Figure 1-15 A typical power pattern polar plot.



Antenna Parameters: Directivity

Directivity: The ratio of the radiation intensity in a certain direction to the average radiation intensity.

$$D(\theta, \phi) \triangleq \frac{U(\theta, \phi)}{U_{ave}(\theta, \phi)} = \frac{U(\theta, \phi) / r^2}{U_{ave}(\theta, \phi) / r^2} = \frac{\frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) \cdot \hat{r}}{P / 4\pi r^2}$$

Radiation intensity: the power radiated in a given direction per unit solid angle

$$U(\theta, \phi) = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) \cdot r^2 \hat{r}$$

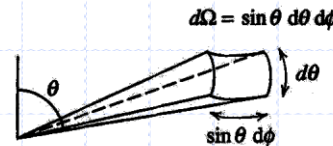


Figure 1-17 Element of solid angle $d\Omega$.

Average radiation intensity:

$$U_{ave}(\theta, \phi) = \frac{1}{4\pi} \iint U(\theta, \phi) d\Omega = P / 4\pi$$

Antenna Parameters: Directivity

When directivity is quoted as a single number without reference to a direction, **maximum directivity** is usually intended.

$$D = \frac{U_m}{U_{ave}} = \frac{4\pi}{\iint |F(\theta, \phi)|^2 d\Omega}$$

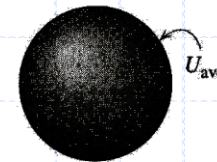
Directivity of Hertzian dipole

$$U(\theta, \phi) = \frac{1}{2} \left(\frac{I \Delta z}{4\pi} \right)^2 \omega \mu \beta \sin^2 \theta$$

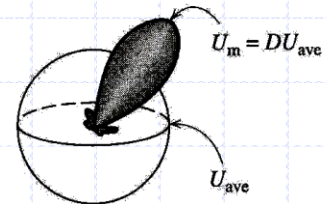
$$\therefore U_m = \frac{1}{2} \left(\frac{I \Delta z}{4\pi} \right)^2 \omega \mu \beta$$

$$U_{ave}(\theta, \phi) = P / 4\pi = \frac{1}{3} \left(\frac{I \Delta z}{4\pi} \right)^2 \beta \omega \mu$$

$$\therefore D = \frac{3}{2} = 10 \log 1.5 = 1.75 \text{ dB}$$



(a) Radiation intensity distributed isotropically



(b) Radiation intensity from an actual antenna

Figure 1-19 Illustration of directivity.



Antenna Parameters: Gain

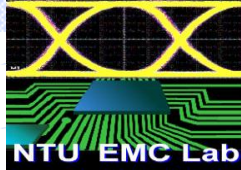
Gain : 4π times the ratio of radiation intensity in a given direction to
The net power accepted by the antenna from the connected transmitter

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{in}}$$

- If all input power appeared as radiated power ($P_{in}=P$), Directivity = Gain.
- In reality, some of the power is lost in the antenna absorbed by the antenna and nearby structures).
- Radiation efficiency:

$$e_r = \frac{P}{P_{in}}, (0 \leq e_r \leq 1)$$

$$\therefore G = e_r D$$



Antenna Parameters: antenna impedance, radiation efficiency

- Input impedance

$$Z_A = R_A + jX_A$$

R_A : Dissipation = Radiation + Ohmic loss

X_A : Power stored near the antenna

$$P_{in} = P + P_{ohmic} = \frac{1}{2} R_r |I_A|^2 + \frac{1}{2} R_{ohmic} |I_A|^2$$

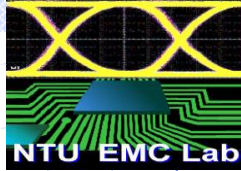
- Efficiency

$$e_r = \frac{P}{P_{in}} = \frac{P}{P + P_{ohmic}} = \frac{R_r}{R_A}$$

- For Ideal dipole

$$R_r = \frac{P}{|I_A|^2} = \frac{2}{I^2} \frac{\omega \mu \beta}{12\pi} (I \Delta z)^2 = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

R_r is very small since $\Delta z \ll \lambda$



Antenna Parameters: ideal dipole is an ineffective radiator

Example

For radiator $\Delta z = 1\text{cm}$, $f_0 = 300\text{MHz}$ ($\lambda = 1\text{m}$) $\rightarrow R_r = 79\text{m}\Omega$

\therefore It needs 3.6A for 1W of radiated power.

For radiator $\Delta z = 1\text{cm}$, $f_0 = 1\text{GHz}$ ($\lambda = 30\text{cm}$) $\rightarrow R_r = 0.87\Omega$

\therefore It needs 1.5A for 1W of radiated power.



Antenna Parameters: How to increase the radiation resistance (efficiency) of the short dipole

Practical short dipole with triangular-like current distribution

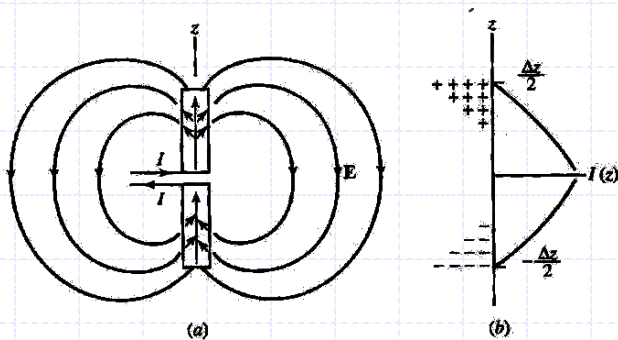
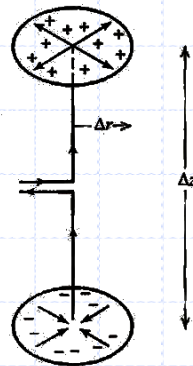


Figure 2-1 Short dipole, $\Delta z \ll \lambda$.
(a) Current on the antenna and the electric fields surrounding it.
(b) Current and charge distributions.

$$R_r = 20\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

1. Capacitor-plate antenna: the top-plate supply the charge such that the current on the wire is constant



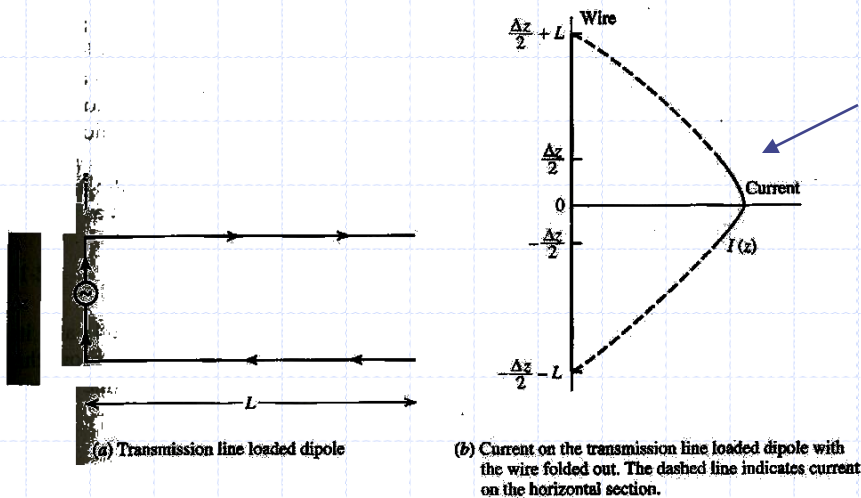
$$R_r = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

Figure 2-3 Capacitor-plate antenna. The arrows on the antenna indicate current. The charges on the plates are also shown.



Antenna Parameters: How to increase the radiation resistance (efficiency) of the short dipole

2. Transmission line loaded antenna



Looks like uniform distribution if $\Delta z \ll L$



(c) The inverted-L antenna

Image theory

3. Inverted-L antenna or Inverted-F antenna

Figure 2-4 Transmission line loaded antennas.



Antenna Parameters: effective aperture

- a. The effective aperture of an antenna, A_e , is the ratio of power received in its load impedance, P_R , to the power density of the incident wave, S_{av} , when the polarization of the incident wave and the polarization of receiving antenna are matched:

$$A_e = \frac{P_R}{S_{av}} \quad (\text{in m}^2)$$

- b. The maximum effective aperture A_{em} is the A_e when the maximum power transferring to the load takes place, which means load impedance is the conjugate to the antenna impedance.



C. Example: compute the A_{em} (maximum effective aperture) of a Hertzian dipole antenna.

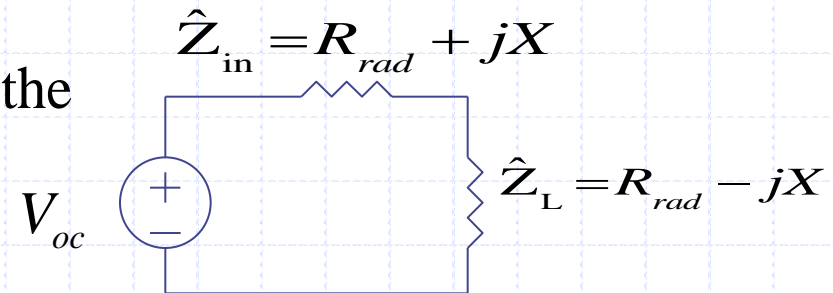
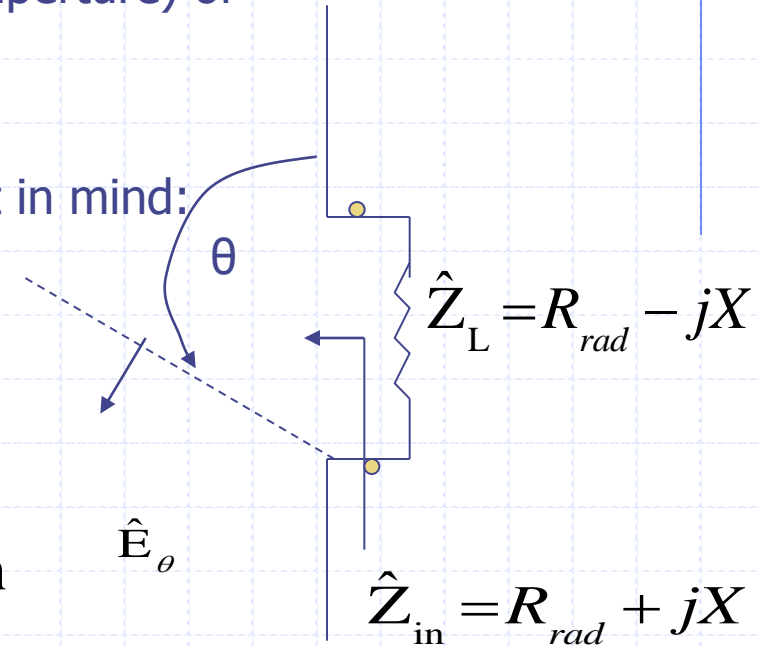
Ans: To solve A_{em} , two conditions should be kept in mind:

- (1) matched polarization for the incident wave and the antenna.
- (2) matched load. (ie. $Z_L = Z_{ant}^*$)

Suppose the incident wave is arriving at an angle θ

(1) the open-circuit voltage produced at the terminals of the antenna is

$$|V_{oc}| = |\hat{E}_\theta| dl \sin \theta$$





(2) power density in the incident wave

$$S_{av} = \frac{1}{2} \frac{|E_{\theta}|^2}{\eta_0}$$

(3) ∴ the load is matched

$$\therefore \text{the received power } P_R = \frac{|V_{oc}|^2}{8R_{rad}} = \frac{|E_{\theta}|^2 dl^2 \sin^2 \theta}{8R_{rad}}$$

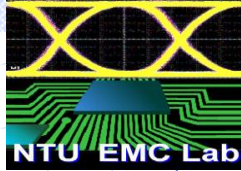
$$\left(\because \frac{1}{2} I^2 R = \frac{1}{2} \left(\frac{V_{oc}}{2R_{rad}} \right)^2 R_{rad} \right)$$

(4) radiation resistance for Hertzian dipole

$$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda_0} \right)^2$$

$$\therefore P_R = \frac{|E_{\theta}|^2 \lambda_0^2}{640\pi^2} \sin^2 \theta$$

$$(5) \therefore A_{em}(\theta, \phi) = \frac{P_R}{S_{av}} = 1.5 \sin^2 \theta \left(\frac{\lambda_0^2}{4\pi} \right) = \frac{\lambda_0^2}{4\pi} D(\theta, \phi)$$



d. For general antennas, the above relation hold.

$$\Rightarrow D(\theta, \phi) = \frac{4\pi}{\lambda_0^2} A_{em}(\theta, \phi) \quad \text{for lossless antenna.}$$

$$\Rightarrow G(\theta, \phi) = \frac{4\pi}{\lambda_0^2} A_{em}(\theta, \phi) \quad \text{for lossy antenna.}$$

The maximum effective aperture of an antenna used for reception is related to the directive gain in the direction of the incoming wave of that antenna when it is used for transmission.



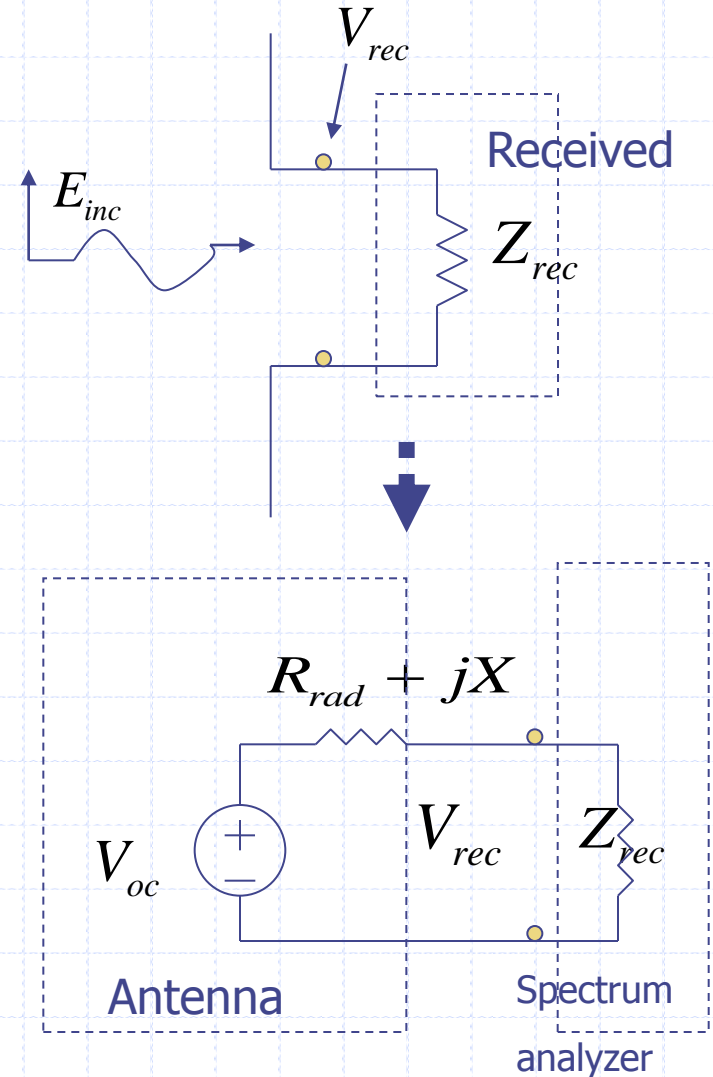
Antenna Factor

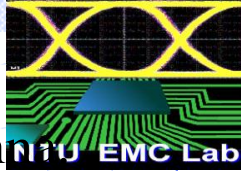
- a. Antenna factor is a common way to characterize the reception properties of EMC antenna.
- b. Antenna factor is defined as the ratio of the incident electric field at the surface of the measurement antenna to the received voltage at the antenna terminals:

$$AF = \frac{|E_{inc}|}{|V_{rec}|}$$

(incident field) (received voltage)

$$\text{Expressed as dB} \Rightarrow AF_{dB} = dB \frac{\mu V}{m} - dB \mu V$$





The antenna factor is usually furnished by the manufacture of the antenna

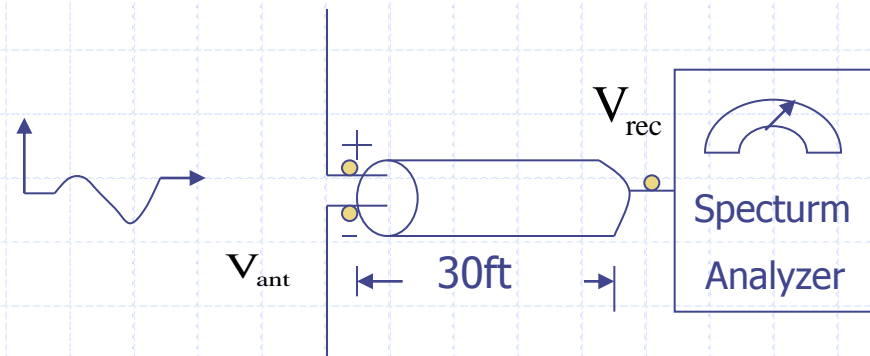
d. Ex: please find the antenna factor at 100MHz for following measurement data.

(1) the field intensity of antenna surface
is 60dBuV/m

(2) The specturm Analyzer measures
40dBuV

(3) coaxial loss=4.5dB/100ft

sol:



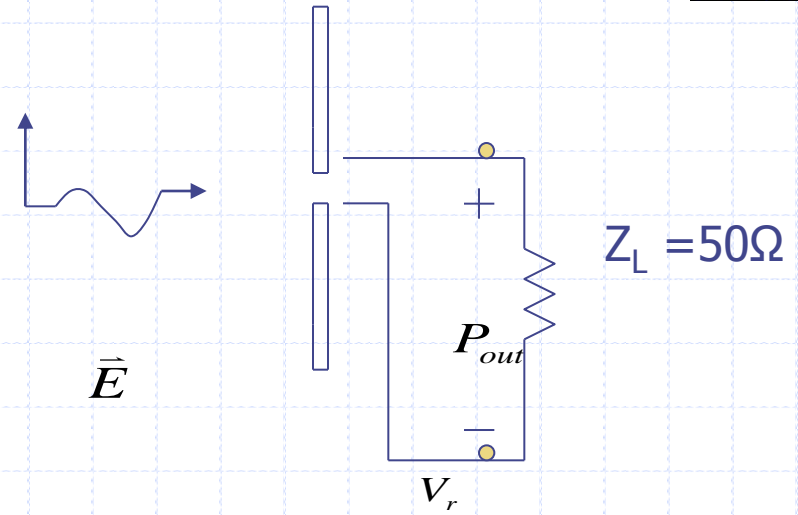
$$\begin{aligned} AF_{dB} &= 60dB \frac{\mu V}{m} - (40dB \mu V + 4.5 \cdot \frac{3}{10} dB) \\ &= 18.65dB \end{aligned}$$



f. $S_{av} = \frac{|\vec{E}|^2}{\eta_0}$

$$\begin{aligned} P_{out} &= S_{av} \cdot A_e \\ &= S_{av} \cdot G_r \cdot \frac{\lambda_0^2}{4\pi} = \frac{V_r^2}{Z_L} \end{aligned}$$

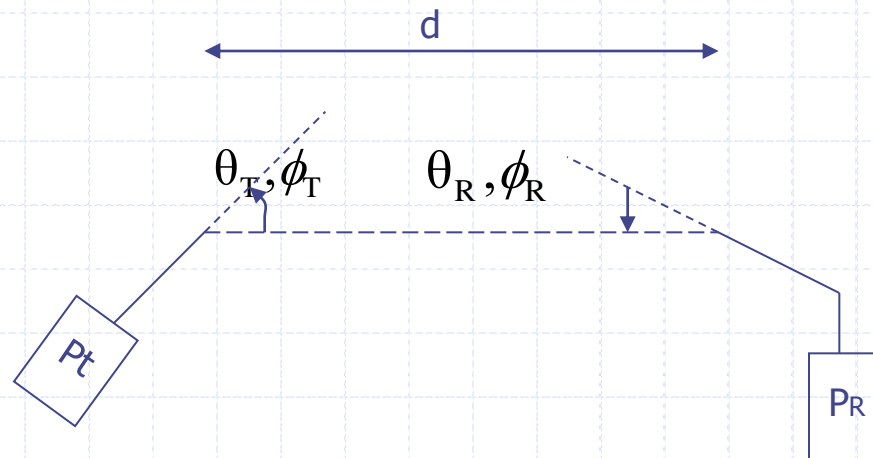
$$\therefore \frac{|\vec{E}|^2}{\eta_0} \cdot G_r \cdot \frac{\lambda_0^2}{4\pi} = \frac{V_r^2}{Z_L} \Rightarrow A.F. = \frac{|\vec{E}|}{|V_r|} = \sqrt{\frac{4\pi\eta_0}{Z_L\lambda^2G_r}}$$





The Friis transmission equation

a. The coupling of two antennas.

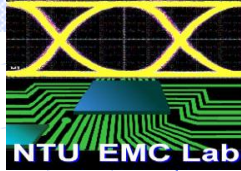


$$G_T(\theta_T, \phi_T)$$

$$A_{eT}(\theta_T, \phi_T)$$

$$G_R(\theta_R, \phi_R)$$

$$A_{eR}(\theta_R, \phi_R)$$



b. Friss transmission equation:

(1) The power density at the receiving antenna of distance d .

$$S_{av} = \frac{P_T}{4\pi d^2} G_T(\theta_T, \phi_T)$$

(2) and by the definition of effective asperture, the received power P_R

$$P_R = S_{av} A_{eR}(\theta_R, \phi_R)$$

$$\therefore \frac{P_R}{P_T} = \frac{G_T(\theta_T, \phi_T) A_{eR}(\theta_R, \phi_R)}{4\pi d^2}$$

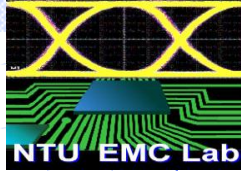
(3) and we know

$$G_R(\theta_R, \phi_R) = \frac{4\pi}{\lambda^2} A_{eR}(\theta_R, \phi_R)$$

$$\therefore \frac{P_R}{P_T} = G_T(\theta_T, \phi_T) \cdot G_R(\theta_R, \phi_R) \cdot \left(\frac{\lambda_0}{4\pi d}\right)^2$$

(4) In terms of dB

$$10\log\left(\frac{P_R}{P_T}\right) = G_{T,dB} + G_{R,dB} - 20\log f - 20\log d + 147.6$$



c. *Note* :

⊕ Friss transmission equation is valid under two assumptions:

(1) the receiving antenna must be matched to its load impedance and the polarization of the incoming wave.

(2) two antenna should be in the far field

i.e. $d > 2D^2 / \lambda_o$ (for surface antennas)

$d > 3\lambda_o$ (for wire antennas)

Half-wave Dipole Antenna:

$$I(z) = I_m \sin[\beta(\frac{\lambda}{4} - |z|)]$$

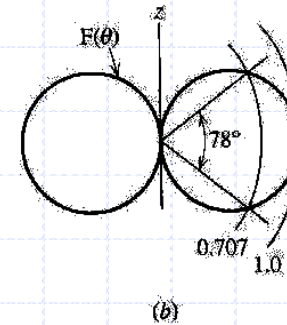
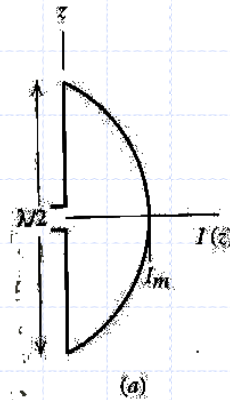


Figure 2-5 The half-wave dipole.
(a) Current distribution $I(z)$.
(b) Radiation pattern $F(\theta)$.

$$\begin{aligned} E_{\theta} &= j\omega\mu \sin\theta \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z' \cos\theta} dz' \\ &= j\omega\mu \frac{2I_m}{\beta} \frac{e^{-j\beta r}}{r} \sin\theta \frac{\cos[(\pi/2) \cos\theta]}{\sin^2\theta} \end{aligned}$$

$$F(\theta) = \frac{\cos[(\pi/2) \cos\theta]}{\sin\theta}$$

Input impedance = $73 + j42.5\Omega$

If the length is slightly reduced to achieve resonance,
the input impedance is about $73 + 0\Omega$



Half-wave Dipole Antenna: performance comparison between some dipoles

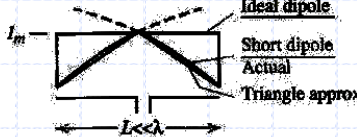

Dipole Type	Length	Current	Pattern	HP	D	D (dB)	R_r (Ω)	R_{ohmic} (Ω)	Current Distribution
Ideal	$L \ll \lambda$	Uniform	$\sin \theta$	90°	1.5	1.76	$80\pi^2 \left(\frac{L}{\lambda}\right)^2$	$\frac{R_s}{2\pi a} L$	
Short	$L \ll \lambda$	Triangle	$\sin \theta$	90°	1.5	1.76	$20\pi^2 \left(\frac{L}{\lambda}\right)^2$	$\frac{R_s}{2\pi a} \frac{L}{3}$	
Half-wave	$L = 0.5 \lambda$	Sinusoid	$\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta}$	78°	1.64	2.15	~ 70	$\frac{R_s}{2\pi a} \frac{\lambda}{4}$	

Figure 2-6 Characteristics and performance of some dipole antennas.



Dipole Antenna: length v.s. directivity

In-phase and enhancement in the far-field

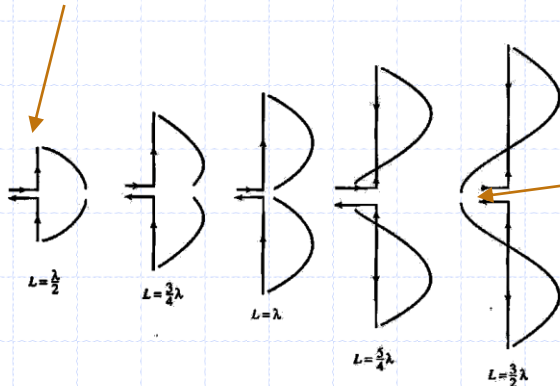


Figure 5-3 Current distributions for various center-fed dipoles. Arrows indicate relative current directions for these maximum current conditions.

Out-of-phase and Cancellation in the far field

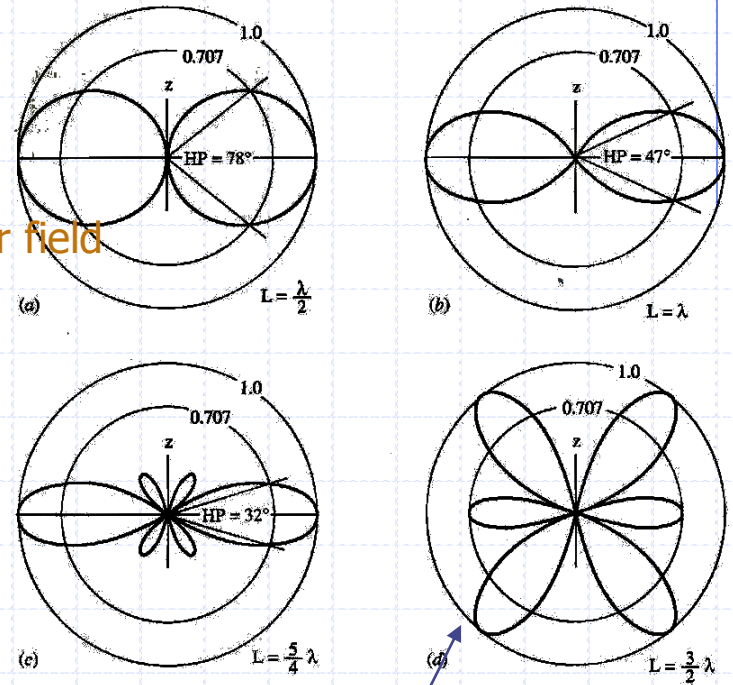


Figure 5-4 Radiation patterns of center-fed straight dipole antennas of length L .

Type	Directivity	HP width
Short dipole	1.5	78°
A $\lambda/2$ dipole	1.64	47°
A λ dipole	2.41	32°
longer	↓	

Directivity is dropped

Dipole Antenna: length v.s. radiation resistance

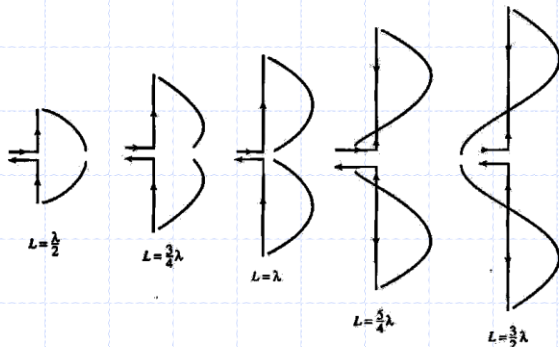


Figure 5-3 Current distributions for various center-fed dipoles. Arrows indicate relative current directions for these maximum current conditions.

Table 5-1 Simple Formulas for the Input Resistance of Dipolas

Length L	Input Resistance (R_{in}), Ω
$0 < L < \frac{\lambda}{4}$	$20\pi^2 \left(\frac{L}{\lambda}\right)^2$
$\frac{\lambda}{4} < L < \frac{\lambda}{2}$	$24.7 \left(\pi \frac{L}{\lambda}\right)^{2.4}$
$\frac{\lambda}{2} < L < 0.637\lambda$	$11.14 \left(\pi \frac{L}{\lambda}\right)^{4.17}$

Input resistance is near infinity because the input current is zero at the feeding point.

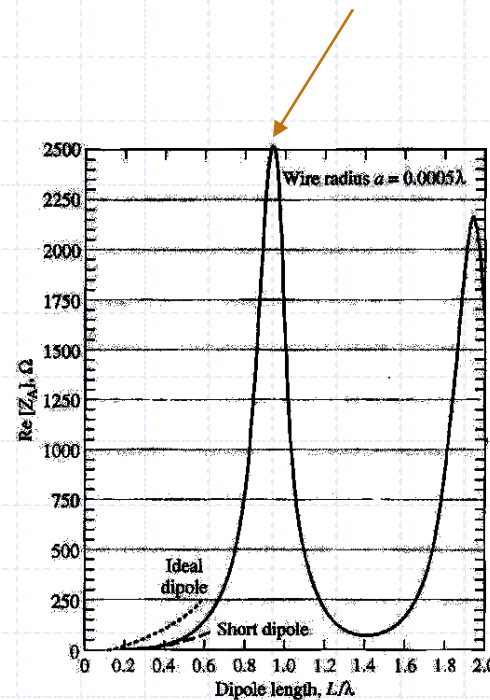


Figure 5-5 Calculated input resistance of a center-fed wire dipole of 0.0005λ radius as a function of length L (solid curve). Also shown is the input resistance $R_{in} = 80\pi^2(L/\lambda)^2$ of an ideal dipole with a uniform current distribution (dotted curve) and the input resistance $R_{in} = 20\pi^2(L/\lambda)^2$ of a short dipole with a triangular current distribution approximation (dashed curve).

Monopole

1. Radiation resistance is half
2. Directivity is double

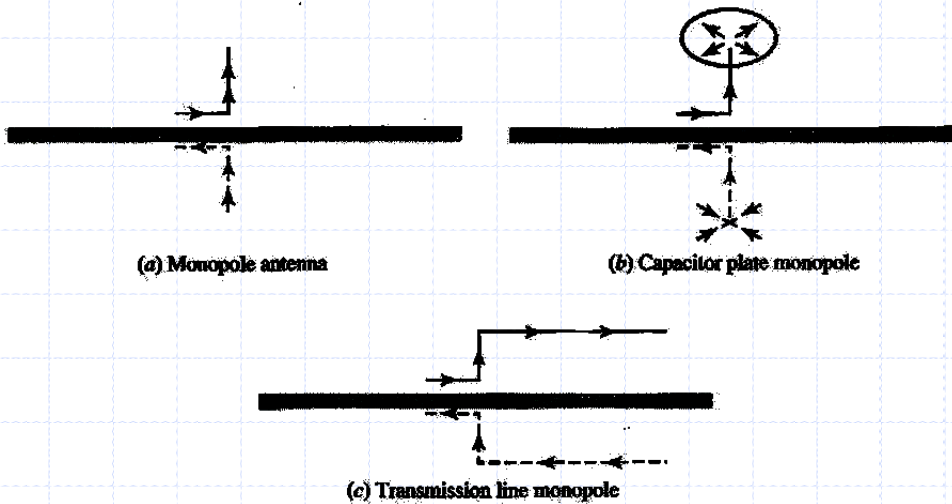


Figure 2-11 Monopole antennas over perfect ground planes with their images (dashed).

$$Z_A = \frac{V_{A,mono}}{I_{A,mono}} = \frac{\frac{1}{2} V_{A,dipole}}{I_{A,dipole}} = \frac{1}{2} Z_{A,dipole}$$

$$D_{mono} = 2D_{dipole}$$

EMI due to Common-mode current can be explained by
The theory of monopole antenna.



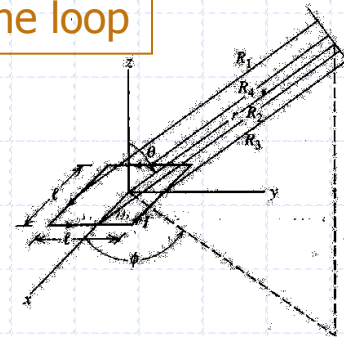
Small loop antenna:

$$\vec{A} = j \frac{\mu I e^{-j\beta r}}{4\pi r} \sin \theta \hat{\phi}$$

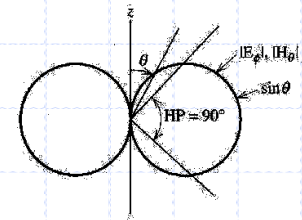
$$\vec{E} = \eta \beta^2 S \frac{I e^{-j\beta r}}{4\pi r} \sin \theta \hat{\phi}$$

$$\vec{H} = \frac{1}{\eta} \hat{r} \times \vec{E} = -\eta \beta^2 S \frac{I e^{-j\beta r}}{4\pi r} \sin \theta \hat{\theta}$$

S is the area of the loop



(a) Geometry for a square loop.
Figure 2-16 The small loop antenna.



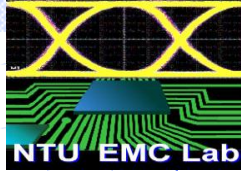
(b) Small loop radiation pattern.

The fields depend only on the magnetic moment (IS) and not the loop shape.

$$R_r = 2P / I^2 = 20(\beta^2 S)^2 \cong 31200 \left(\frac{S}{\lambda^2} \right)^2 \Omega$$

The radiation resistance can be increased by using multiple turns (N)

$$R_r = 31200 \left(\frac{NS}{\lambda^2} \right)^2 \Omega$$



Small loop antenna:

- The small loop antenna is very popular in receiving antenna.
- Single turn small loop antennas are used in paggers.
- Multi-turn small loops are popular in AM broadcast receiver.
- Small loop antennas are also used in direction-finding receivers and for EMI field strength probes.



Yagi-Uda Antenna

- ◆ Array Antenna can be used to increase directivity.
- ◆ Array feed networks are considerably simplified if only a few elements are fed directly. Such array is referred to as a parasitic array.
- ◆ A parasitic linear array of parallel dipole is called a Yagi-Uda antenna.
- ◆ Uda antennas are very popular because of their simplicity and relatively high gain.



Yagi-Uda Antenna: Principles of operation

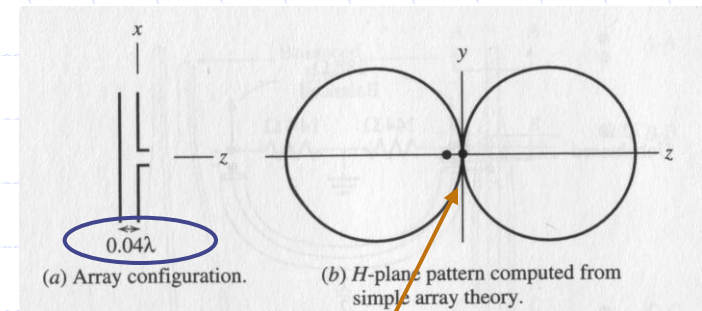
If a parasitic element is positioned very close to the driver element, it is excited with roughly equal amplitude.

$$E_{incident} = E_{driver}$$

$$E_{parasitic} = -E_{incident} = -E_{driver}$$

Because the tangential E field on a good conductor should be zero.

From array theory, we know that the closely spaced, equal amplitude, opposite phase elements will have end-fire patterns.



Why null here?

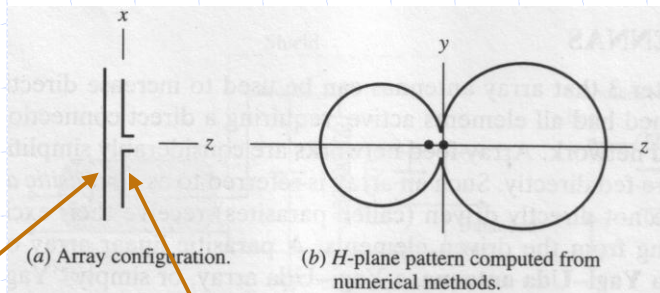


Yagi-Uda Antenna: Principles of operation

The simplistic beauty of Yagi is revealed by **lengthening** the parasite
The dual end-fire beam is changed to a more desirable single end-fire beam.

The H-plane pattern is obtained by the numerical method.

Such a parasite is called a **reflector** because it appears to reflect radiation from the driver.



0.49λ

0.4781λ

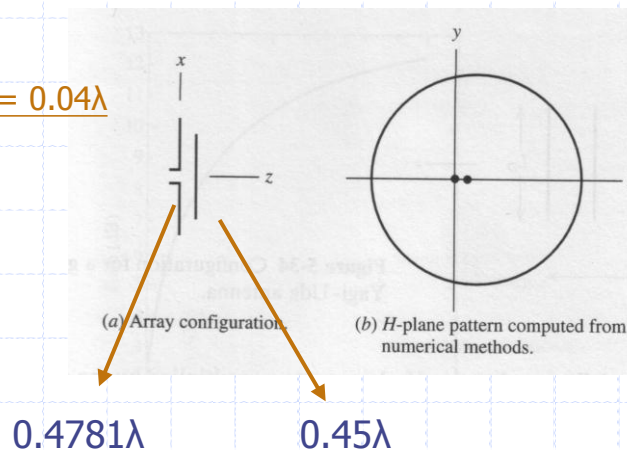


Yagi-Uda Antenna: Principles of operation

If the parasite is shorter than the driver, but now placed on **the other side** of the driver, the pattern effect is similar to that when using a reflector in the sense that main beam enhancement is in the same direction.

This parasite is then referred to as a **director** since it appears to direct radiation in the direction from the driver toward the director.

Spacing = 0.04λ

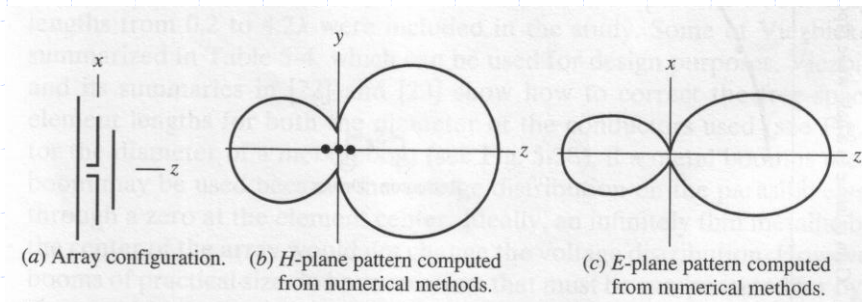




Yagi-Uda Antenna: Principles of operation

The single endfire beam can be further enhanced with a reflector and a director on opposite sides of a driver.

The maximum directivity obtained from three-element Yagi is about 9dBi or 7dBd.





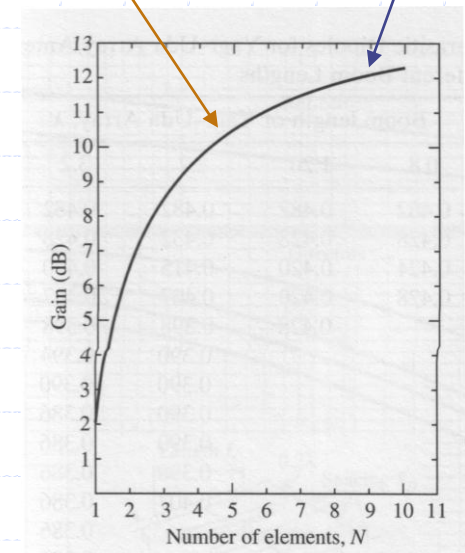
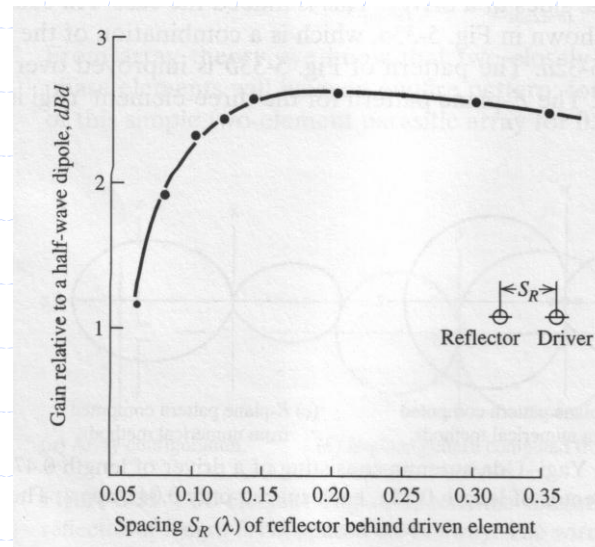
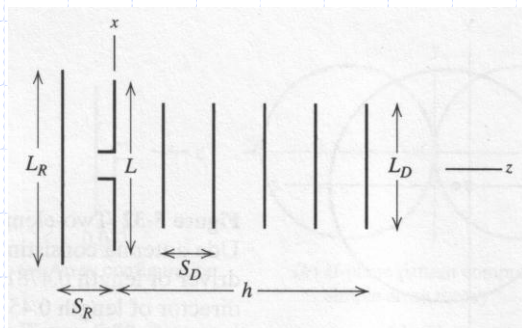
Yagi-Uda Antenna: Principles of operation

Optimum reflector spacing S_R (for maximum directivity) is between **0.15 and 0.25 wavelengths**.

Director-to-director spacings are typically **0.2 to 0.35 wavelengths**, with the larger spacings being more common for long arrays and closer spacings for shorter arrays.

The **director length** is typically 10 to 20% shorter than their resonant length.

The addition of directors up to 5 or 6 provides a significant increase in gain.



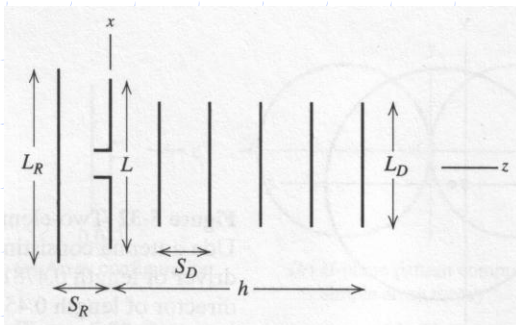


Yagi-Uda Antenna: Principles of operation

Table 5-4 Optimized Lengths of Parasitic Dipoles for Yagi-Uda Array Antennas of Six Different Boom Lengths

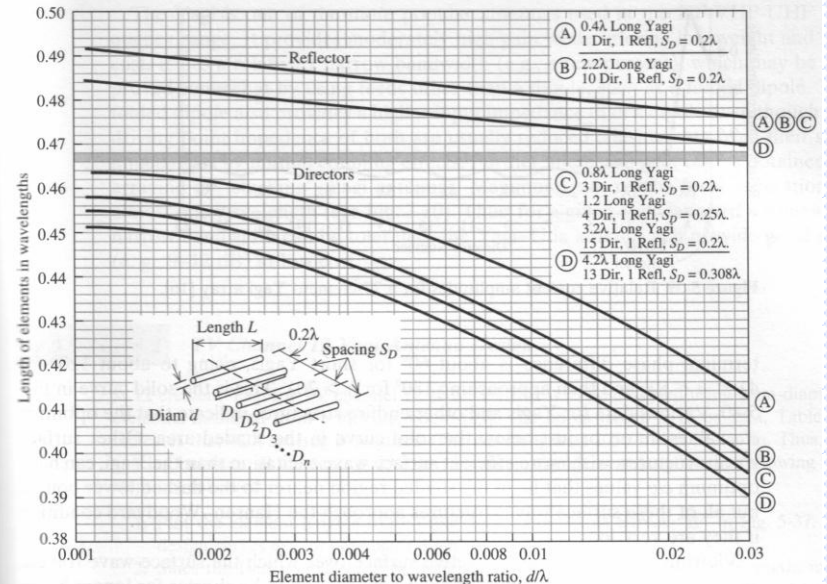
	Boom length of Yagi-Uda Array, λ					
	0.4	0.8	1.20	2.2	3.2	4.2
$d/\lambda = 0.0085$ $S_R = 0.2\lambda$						
Length of reflector, L_R/λ	0.482	0.482	0.482	0.482	0.482	0.475
D_1	0.442	0.428	0.428	0.432	0.428	0.424
D_2		0.424	0.420	0.415	0.420	0.424
D_3		0.428	0.420	0.407	0.407	0.420
D_4			0.428	0.398	0.398	0.407
D_5				0.390	0.394	0.403
D_6				0.390	0.390	0.398
D_7				0.390	0.386	0.394
D_8				0.390	0.386	0.390
D_9				0.398	0.386	0.390
D_{10}				0.407	0.386	0.390
D_{11}					0.386	0.390
D_{12}					0.386	0.390
D_{13}					0.386	0.390
D_{14}					0.386	
D_{15}					0.386	
Spacing between directors (S_D/λ)	0.20	0.20	0.25	0.20	0.20	0.308
Gain relative to half-wave dipole, dBd	7.1	9.2	10.2	12.25	13.4	14.2
Design curve (Fig. 5-37)	(A)	(C)	(C)	(B)	(C)	(D)
Front-to-back ratio, dB	8	15	19	23	22	20

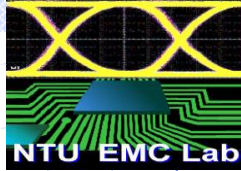
Source: P. P. Vezibicke, "Yagi Antenna Design," NBS Tech. Note 688, National Bureau of Standards, Washington, DC, Dec. 1968.



The main effect of the reflector is on the driving point impedance at the feeding point and on the back lobe of The array.

Pattern shape, and therefore gain, are mostly controlled by The director elements.





Broadband Measurement Antennas

- a. FCC prefers to use tuned half-wave dipoles. For measurement of radiated emission from 30MHz~1GH, it is time consuming because its length must be physically adjusted to provide $\lambda/2$ at each frequency.
- b. Broadband measurement antennas for EMC.
 - 1. Input / output impedance is fairly constant over the frequency band.
 - 2. The pattern is fairly constant over the frequency band.two commonly used antenna
 - 1. biconical antenna -- 30MHz~200MHz
 - 2. log-periodic antenna -- 200MHz~1GHz



The Biconical Antenna

a. Infinite biconical antenna

1. half angle θ_h .
2. feed point with small gap.

b. In the free space , the symmetry

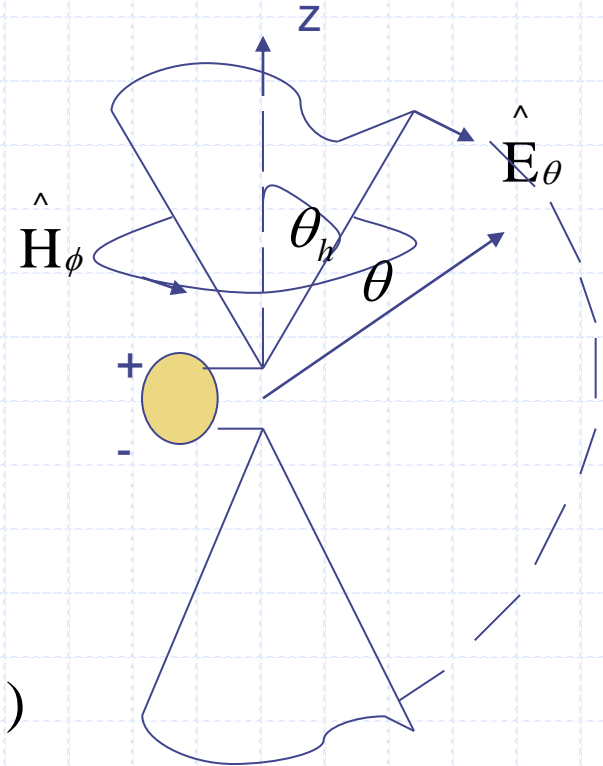
suggests: $\hat{H} = \hat{H}_\phi \hat{a}_\phi$

$$\hat{E} = \hat{E}_\theta \hat{a}_\theta$$

$$\nabla \times \bar{H} = j\omega\epsilon \bar{E} + \bar{J}, \text{ for } \bar{J} = 0$$

$$(1/r \sin \theta) \partial / \partial \theta (\sin \theta H_\phi) = j\omega\epsilon E_r = 0 \dots\dots(1)$$

$$(-1/r) \partial / \partial r (r H_\phi) = j\omega\epsilon E_\theta \dots\dots(2)$$





c. by the Faraday's and Ampere's law

$$\hat{H}_\phi = (H_0 / \sin \theta) e^{-j\beta_0 r} / r$$

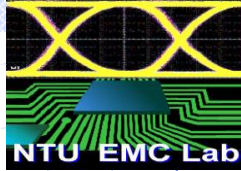
$$\text{from (1)} \Rightarrow \partial H_\phi \sin \theta / \partial \theta = 0$$

$$\text{and } \hat{E}_\theta = (\beta_0 H_0 / \omega \epsilon_0 \sin \theta) e^{-j\beta_0 r} / r$$

from (2) and TEM mode type

$$\Rightarrow \hat{E}_\theta = \eta_0 \hat{H}_\phi$$

so , we may uniquely define voltage between two points on the cores as was the case in tx lines.



d. $\hat{V}(r) = - \int_{\theta=\pi-\theta_h}^{\theta_h} \vec{E} \cdot d\vec{l} = - \int_{\theta=\pi-\theta_h}^{\theta_h} (\eta_0 H_0 e^{-j\beta_0 r} / r \sin \theta) * r d\theta$

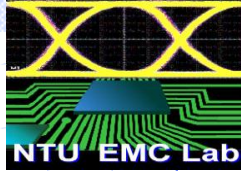
$$= -\eta_0 H_0 e^{-j\beta_0 r} \int_{\pi-\theta_h}^{\theta_h} d\theta / \sin \theta = 2\eta_0 H_0 e^{-j\beta_0 r} \ln (\cot(\theta_h / 2))$$

by Ampere's law $\Rightarrow \hat{I}(r) = \int_{\Phi=0}^{2\pi} \hat{H}_{\Phi} \cdot r \cdot \sin \theta d\phi = 2\pi H_0 e^{-j\beta_0 r}$

e. Input impedance : (at $r = 0$)

$$\hat{Z}_{in} = \hat{V}(r) / \hat{I}(r) \Big|_{r=0} = (\eta_0 / \pi) \ln(\cot(\theta_h / 2)) = 120 \ln(\cot(\theta_h / 2))$$

Usually , the half angle is choosen such that $\hat{Z}_{in} = \hat{Z}_C$



f. for infinite biconnical antenna

$$R_{\text{rad}} = \hat{Z}_{\text{in}}$$

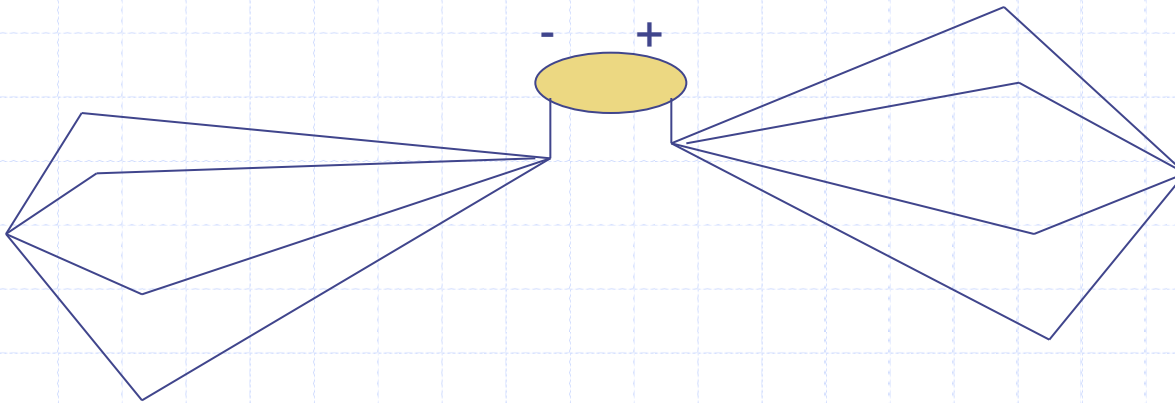
$$\begin{aligned} \text{proof} \Rightarrow P_{\text{rad}} &= \oint_S \overrightarrow{S_{\text{av}}} d\overrightarrow{s} = \int_0^{2\pi} \int_{\theta=\theta_h}^{\pi-\theta_h} \left| \hat{E}_{\theta} \right|^2 / 2\eta_0 \cdot r^2 \sin \theta d\theta d\phi \\ &= \pi \eta_0 H_0^2 * \int_{\theta=\theta_h}^{\pi-\theta_h} d\theta / \sin \theta = 2\pi \eta_0 |H_0|^2 * \ln(\cot(\theta_h / 2)) \\ &= (1/2)(2\pi H_0)^2 (\eta_0 / \pi * \ln(\cot(\theta_h / 2))) \\ &= (1/2)(\hat{I})^2 R_{\text{rad}} = (1/2)(\hat{I})^2 \hat{Z}_{\text{in}} \end{aligned}$$



- f. note : (1) \hat{Z}_{in} is independent of frequency (flat response) for infinite core
(2) but , practice antenna is finite length , so

$$\hat{Z}_{in}' = R + j\underline{X} \rightarrow \text{some storage energy will appear}$$

g. EMC biconical antenna



With wire element

Log-Periodic Dipole Array (LPDA) Antenna

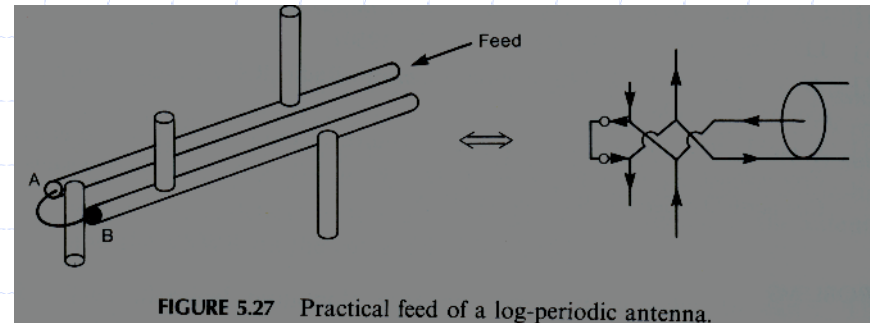
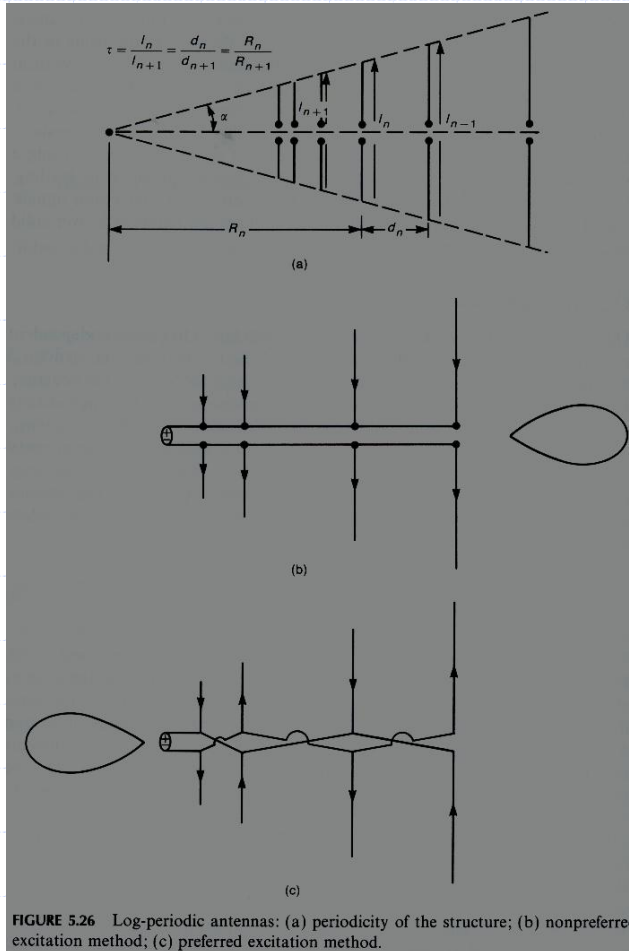
Scale factor

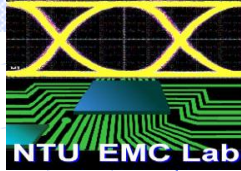
$$\tau = \frac{R_{n+1}}{R_n} < 1$$

Spacing factor

$$\sigma = \frac{d_n}{2L_n}$$

Why is the LPDA constructed as below?



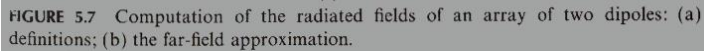
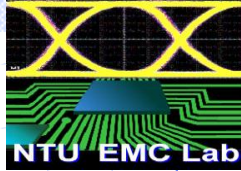


Log-Periodic Dipole Array (LPDA) Antenna

It is convenient to view the LPDA operation as being similar to that of a Yagi-Uda antenna.

The longer dipole behind the most active dipole behaves as a reflector and the adjacent shorter dipole in front acts as a director.

The radiation is then off of the apex.



- Hertzian dipole, small loop, half dipole, and monopole are all **omnidirectional** because of symmetry of the structure.
- Two or more omni-directional antennas can **change the pattern** (create null or maximum). Better both for **communication** and **EMI** considerations.
- This can be achieved by **phasing the currents** to the antennas and separated them sufficiently such that the fields will add **constructively and destructively** to result in the null/maximum patterns.

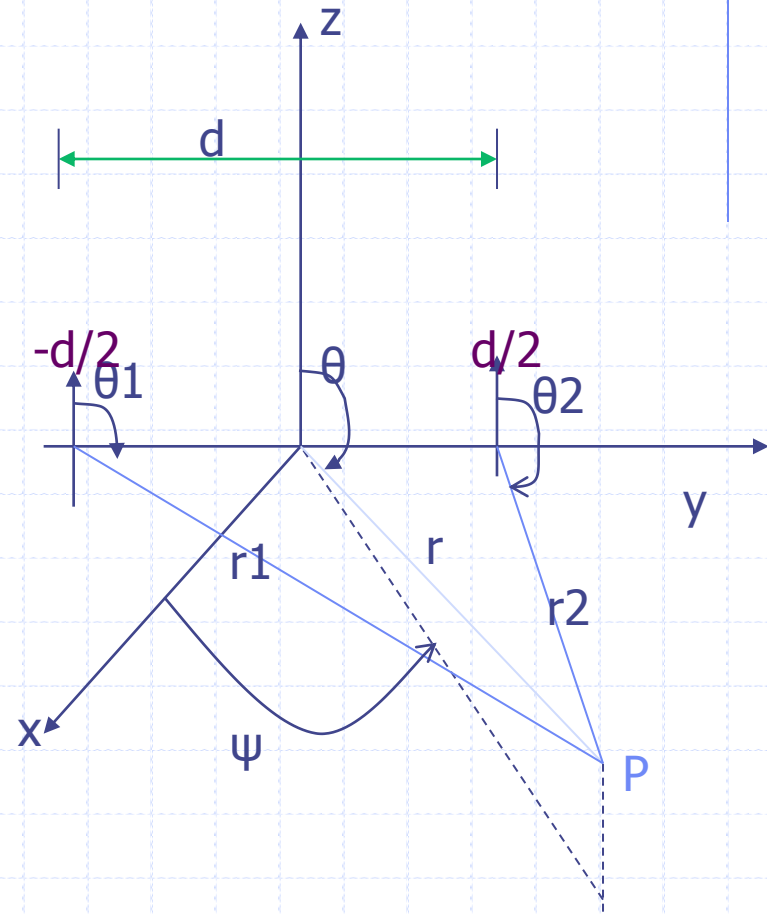


Antenna arrays

- a. By using two or more omni-directional antennas to produce maxima and/or nulls in the resulting pattern.
- b. The far fields at point P due to each antenna are of the form:

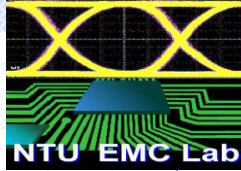
$$\hat{E}_{\theta_1} = \frac{\hat{M}I \angle \alpha}{r_1} e^{-j\beta_0 r_1}$$

$$\hat{E}_{\theta_2} = \frac{\hat{M}I \angle 0}{r_2} e^{-j\beta_0 r_2}$$





note:



1. $I_1 = I \angle 0$ and $I_2 = I \angle \alpha$, we assume that the currents of the two antennas are equal in magnitude but with phase difference α .

2. \hat{M} depends on the type of antennas used.

for Hertzian dipole: $\hat{M} = j\eta_0\beta_0 \left(\frac{dl}{4\pi} \right) \sin \theta$

for long dipole : $\hat{M} = j\varepsilon_0 F(\theta)$.

c. The total field at point P

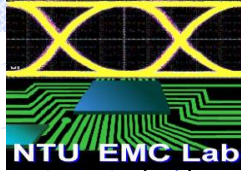
$$\hat{E}_\theta = \hat{E}_{\theta 1} + \hat{E}_{\theta 2} = \hat{M} I e^{\frac{j\alpha}{2}} \left(\frac{e^{-j\beta_0 r_1}}{r_1} e^{\frac{j\alpha}{2}} + \frac{e^{-j\beta_0 r_2}}{r_2} e^{\frac{-j\alpha}{2}} \right)$$

using for field approximation.

(1) $r_1 \cong r_2 \cong r$ for distance term.

$$(2) r_1 \cong r + \frac{d}{2} \cos \gamma \cong r + \frac{d}{2} \sin \theta \sin \phi$$

$$r_2 \cong r - \frac{d}{2} \cos \gamma \cong r - \frac{d}{2} \sin \theta \sin \phi \quad , \text{where } \cos \gamma = \bar{a}_r \cdot \bar{a}_y = \sin \theta \sin \phi$$



$$\begin{aligned} \hat{E}_\theta &= \frac{\hat{M}I}{r} e^{\frac{j\alpha}{2}} e^{-j\beta_0 r} \left[e^{j\left(\beta_0 \left(\frac{d}{2}\right) \sin \theta \sin \phi - \frac{\alpha}{2}\right)} + e^{-j\left(\beta_0 \left(\frac{d}{2}\right) \sin \theta \sin \phi - \frac{\alpha}{2}\right)} \right] \\ &= 2e^{\frac{j\alpha}{2}} \left[\hat{M} \frac{Ie^{-j\beta_0 r}}{r} \right] \times \cos \left(\frac{\pi d}{\lambda_0} \sin \theta \sin \phi - \frac{\alpha}{2} \right) \end{aligned}$$

pattern of individual elements $F_{array}(\theta, \phi)$

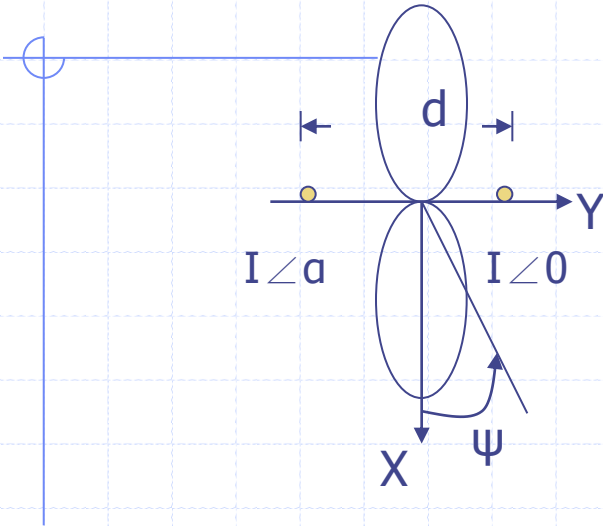
d. The principle of Pattern Multiplication:

The resultant field is the product of the individual antenna element and the array factor $F_{array}(\theta, \phi)$.

e. Some examples of the patterns of two-element array

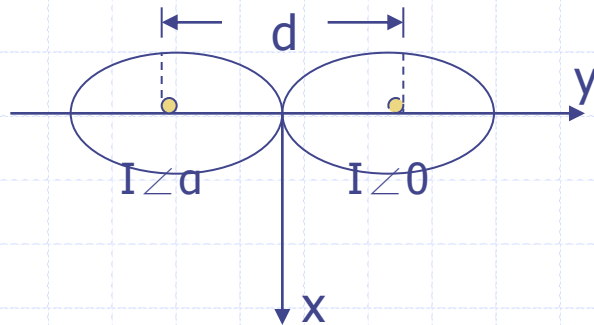


(1) $d=\lambda/2, a=0$



$$F_{\text{array}}(\theta=90^\circ, \phi) = \cos\left(\frac{\pi \sin \phi}{2}\right)$$

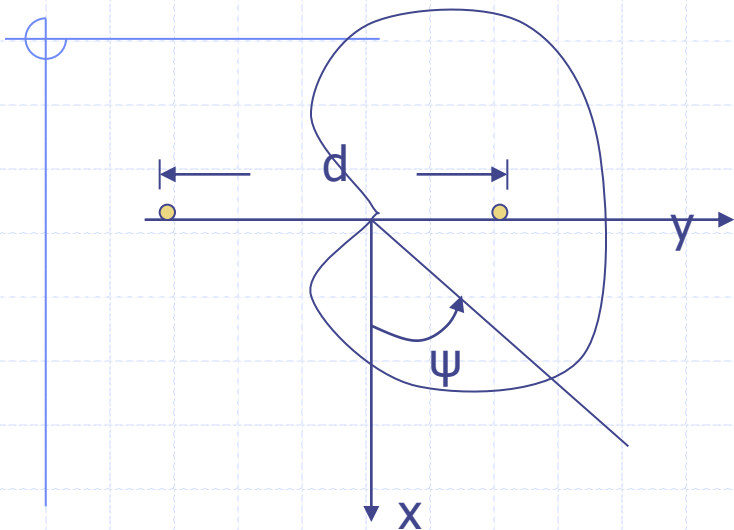
(2) $d=\lambda/2, a=\pi$



$$F_{\text{array}}(\theta=90^\circ, \phi) = \cos\left(\frac{\pi \sin \phi}{2} - \frac{\pi}{2}\right)$$



(3) $d=\lambda/4, a=n/2$



$$F_{\text{array}}(\theta=90^\circ, \phi) = \cos\left(\frac{\pi \sin \phi}{4} - \frac{\pi}{4}\right)$$

Antenna Arrays:

- Two arbitrary omni-directional antennas
- Separate a distance d
- currents on two antennas are equal in amplitude and phase delay α

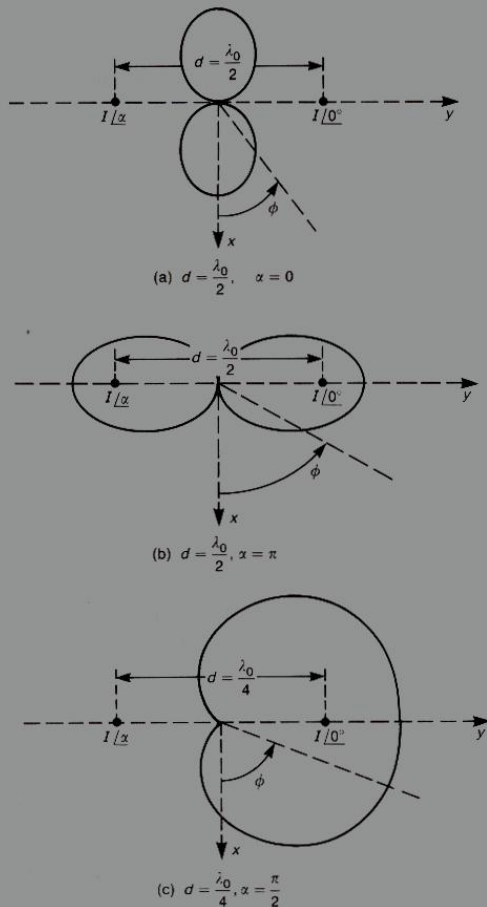
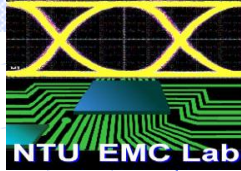


FIGURE 5.8 Patterns of a two-element array: (a) $d = \frac{1}{2}\lambda_0, \alpha = 0^\circ$; (b) $d = \frac{1}{2}\lambda_0, \alpha = 180^\circ$; (c) $d = \frac{1}{4}\lambda_0, \alpha = 90^\circ$.

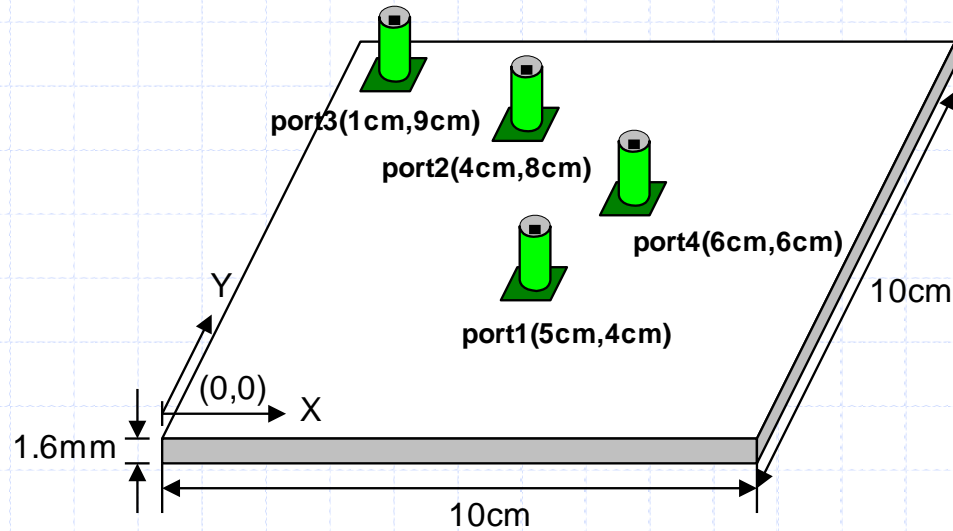
$$E_\theta = 2e^{j\alpha/2} \left[M \frac{Ie^{-j\beta_0 r}}{r} \right] \times \cos\left(\frac{\pi d}{\lambda_0} \sin \theta \sin \phi - \frac{\alpha}{2}\right)$$

Pattern of individual antenna

Array pattern



Microstrip Antenna and Resonance frequency

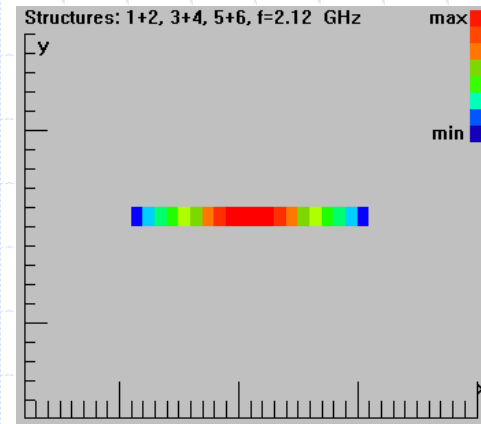
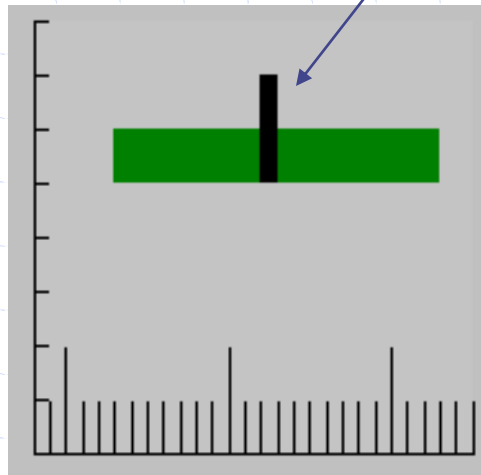


$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$



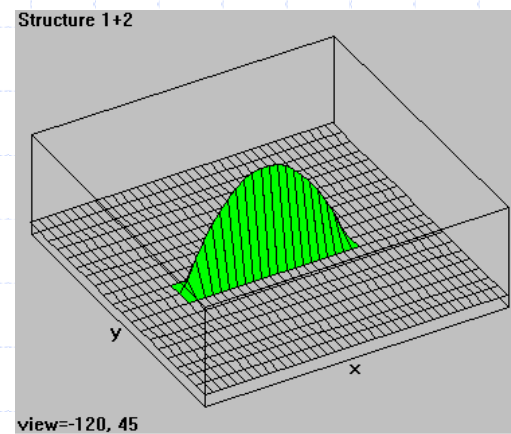
Antenna Examples: Printed Dipole (at 2.12 GHz)

Source excitation



2D

Current distribution

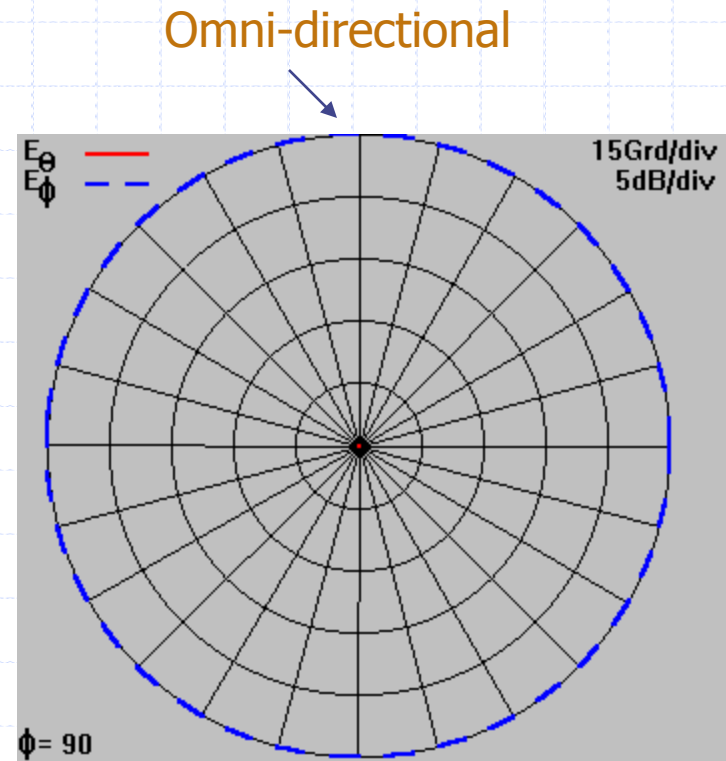
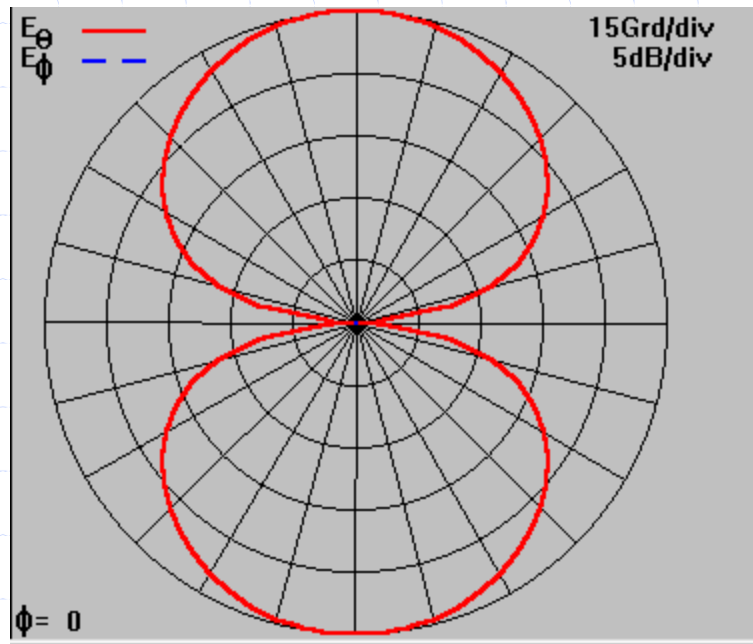


3D

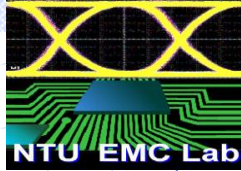


Antenna Examples: Printed Dipole

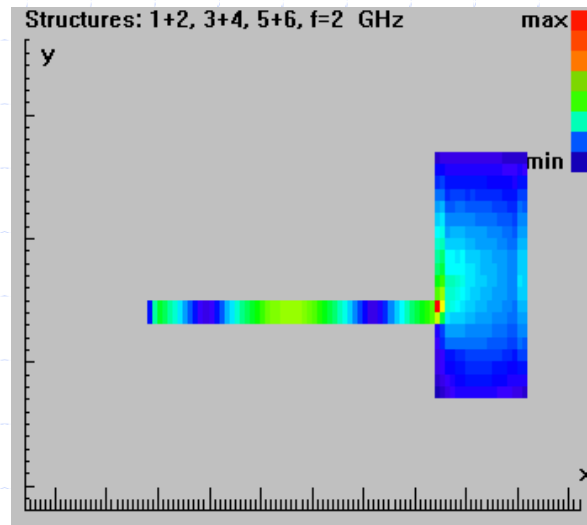
Radiation Patterns



Gain: 2.1dB
Efficiency : 99.1%



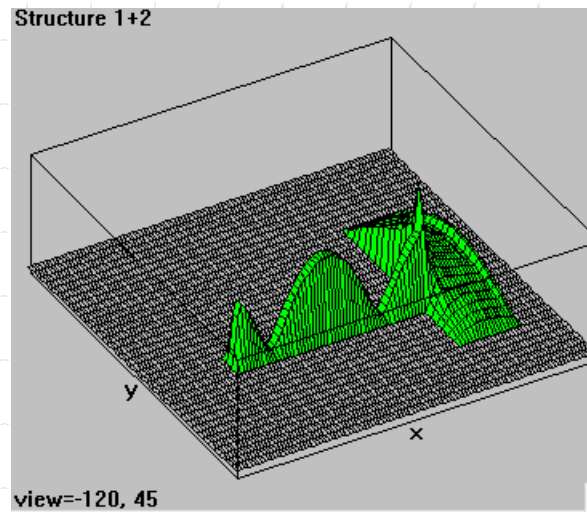
Antenna Examples: Microstrip Antenna with edge coupled (at 2.1GHz)



2D

Current distribution

3D

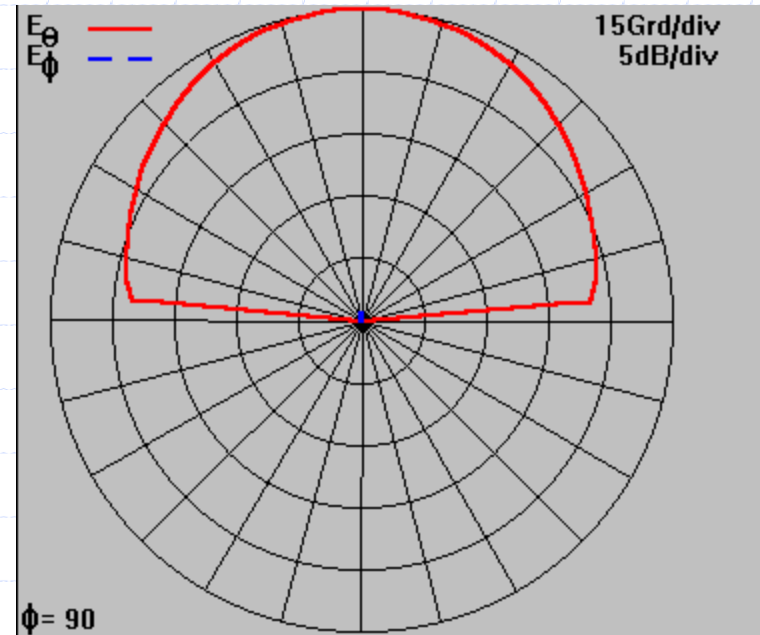
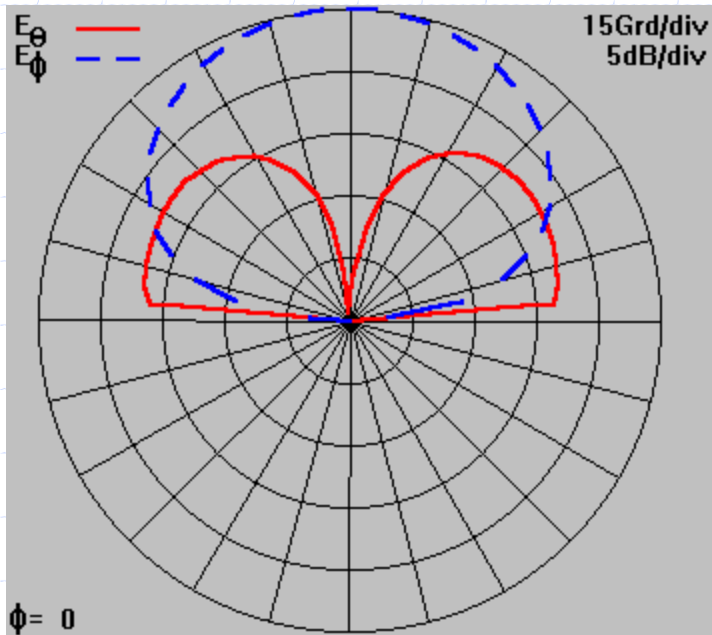


3cm x 4cm

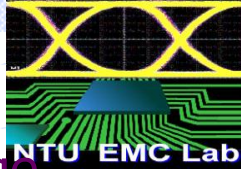
$\epsilon=2.2$



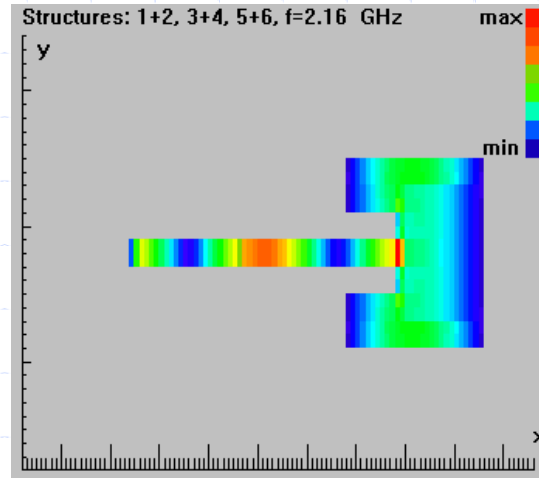
Antenna Examples: Microstrip Antenna with edge coupled (at 2.1GHz)



Gain: 6.6dB
Efficiency: 71.1%



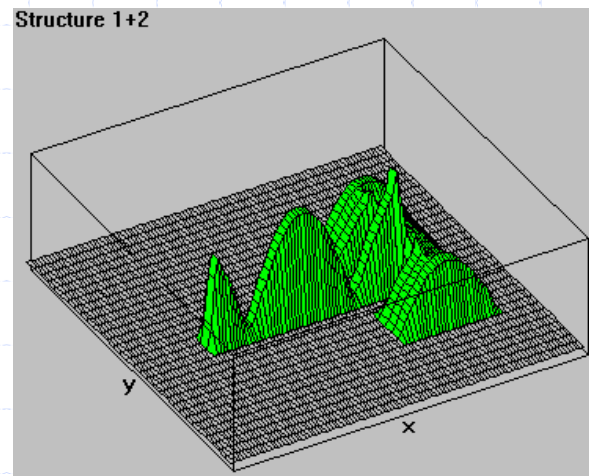
Antenna Examples: Microstrip Antenna with inserted feed and edge coupled (at 2.1GHz)



2D

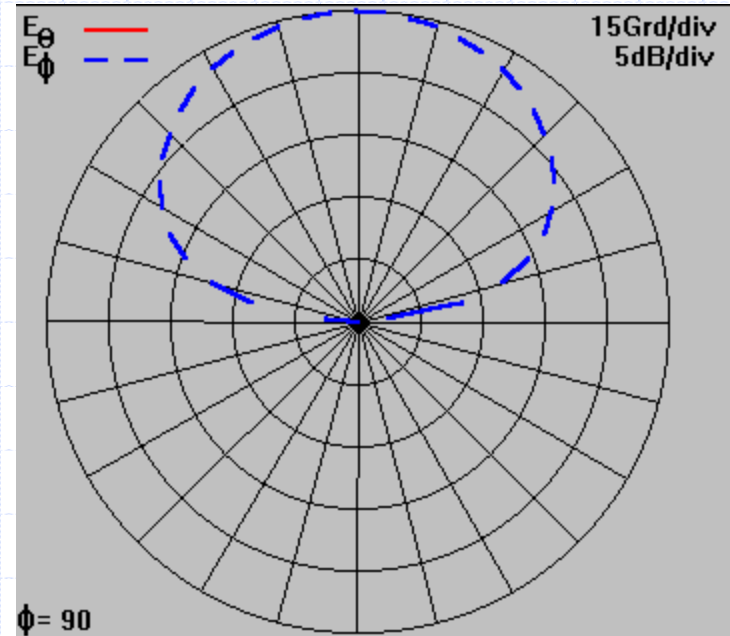
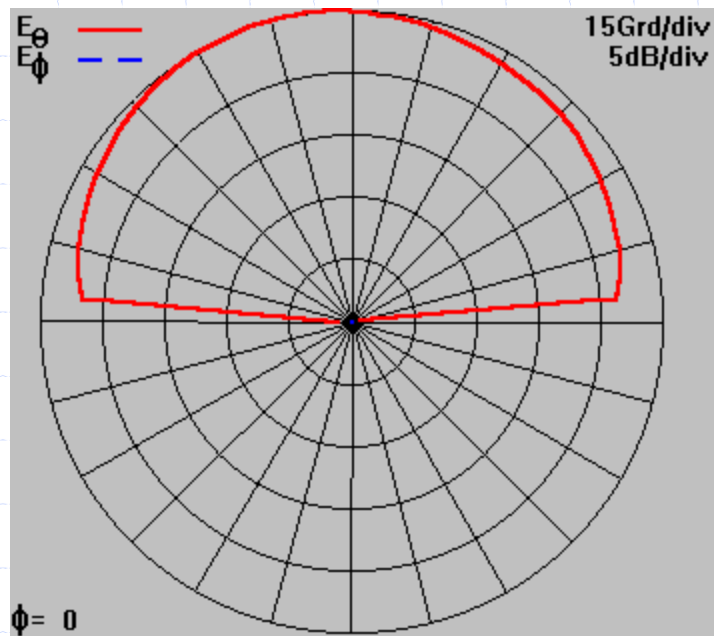
Current distribution

3D





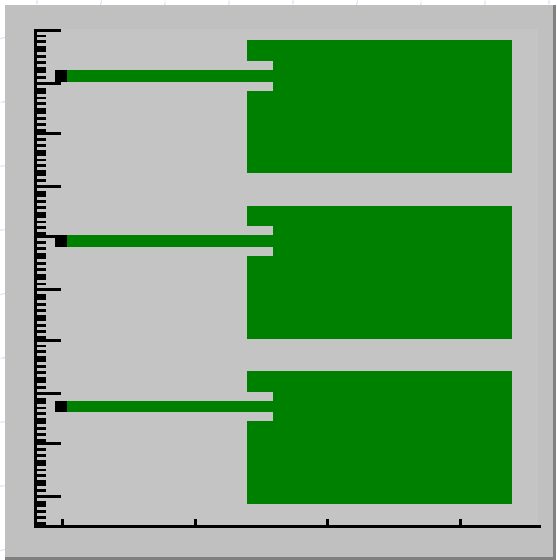
Antenna Examples: Microstrip Antenna with inserted feed and edge coupled (at 2.1GHz)



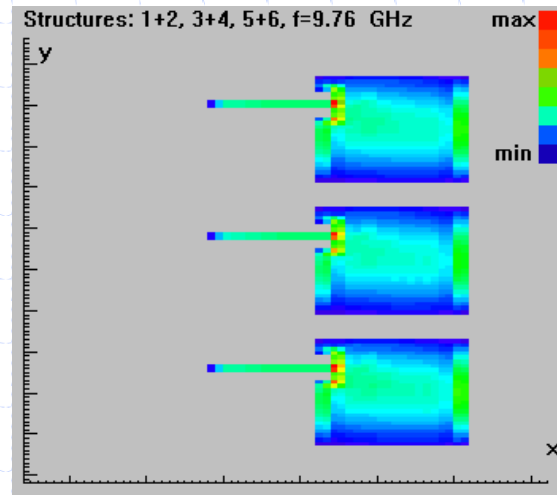
Gain: 6.4dB
Efficiency: 49.1%



Antenna Examples: Small Array (at 9.76 GHz)

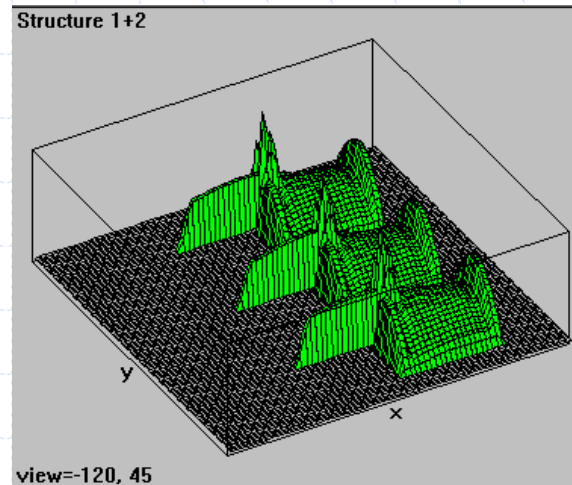


X = 7.5mm
Y = 3.75mm



2D

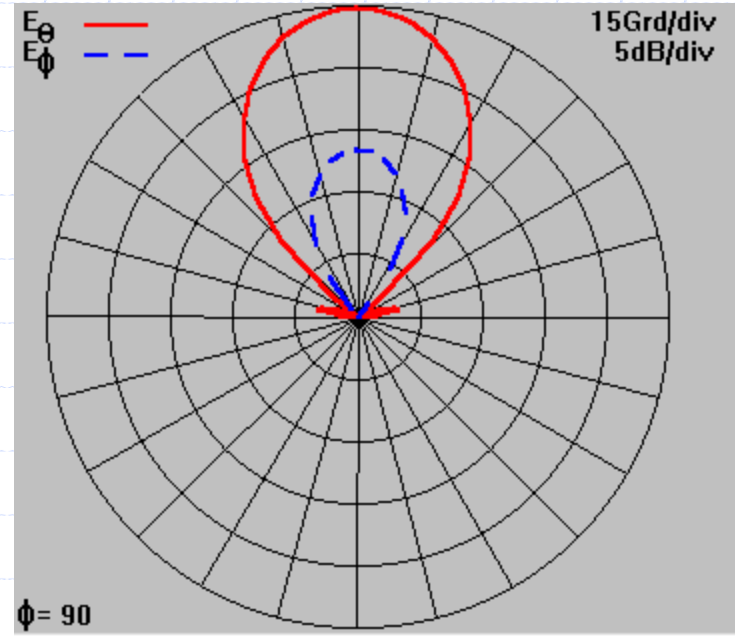
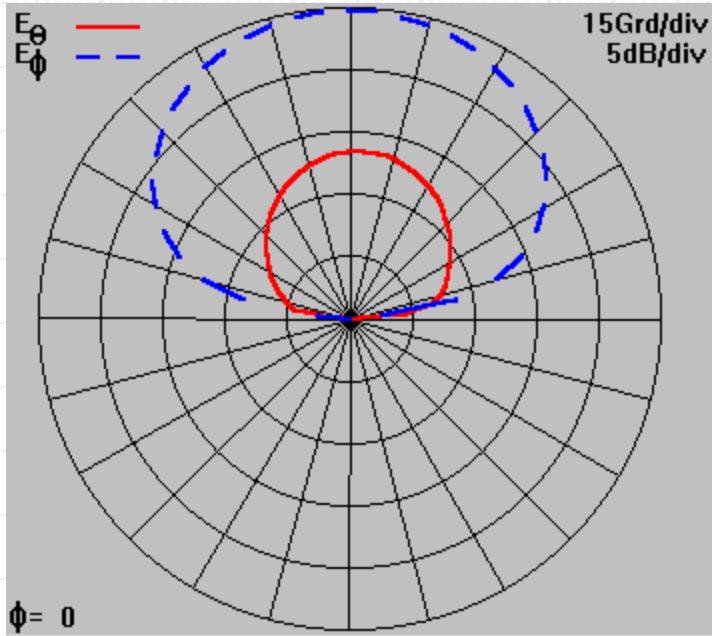
Current distribution



3D



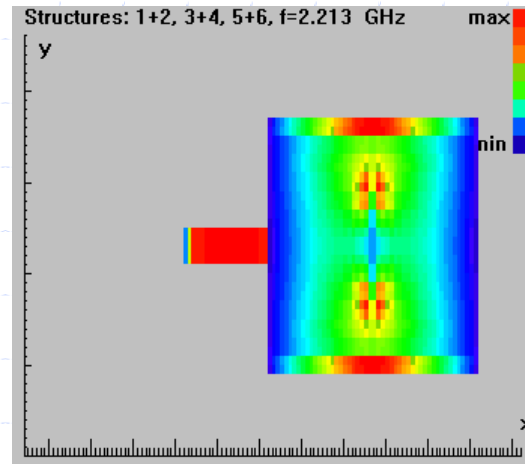
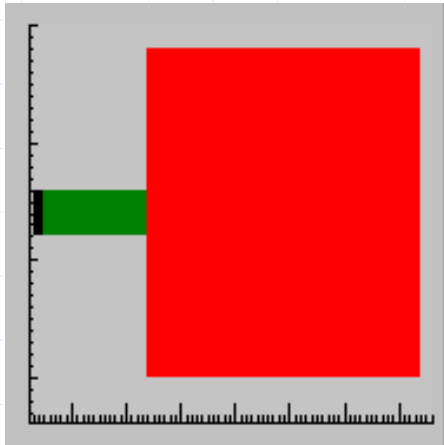
Antenna Examples: Small Array (at 9.76 GHz)



Gain: 10.5dB

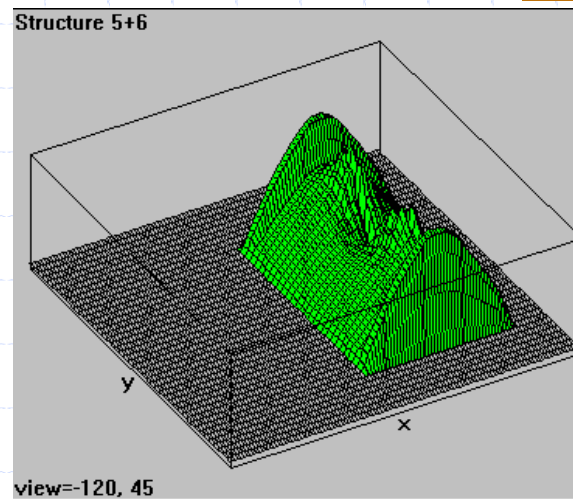
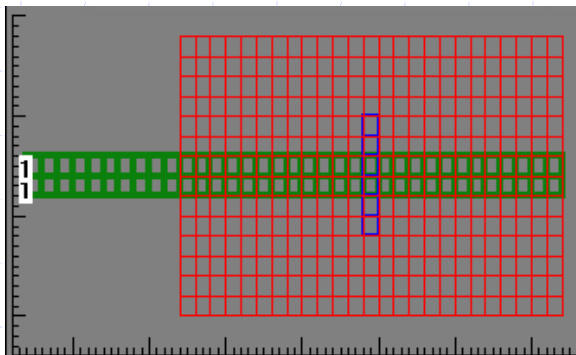


Antenna Examples: slot coupled microstrip antenna (at 2.213 GHz)



2D

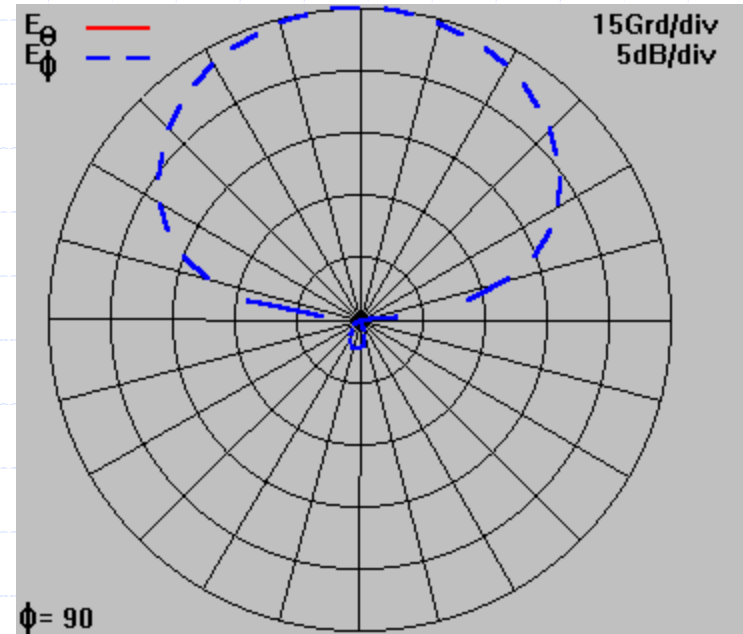
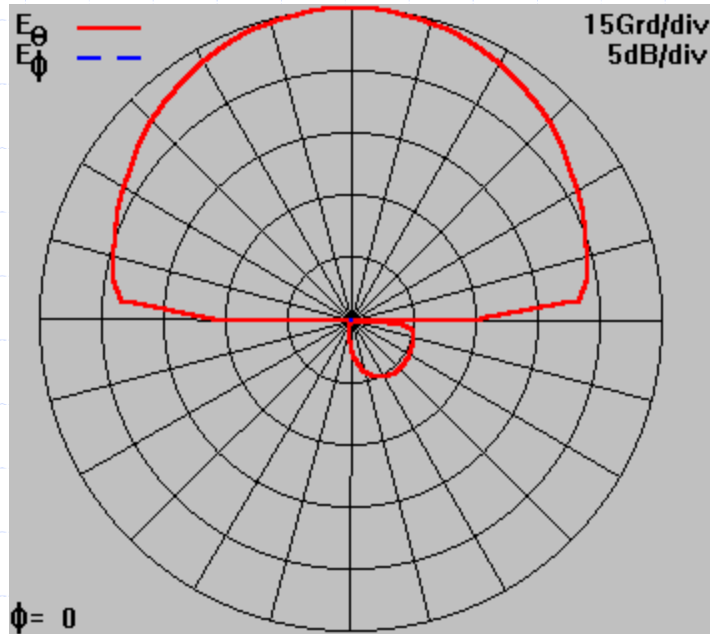
Current distribution



3D



Antenna Examples: slot coupled microstrip antenna (at 2.213 GHz)



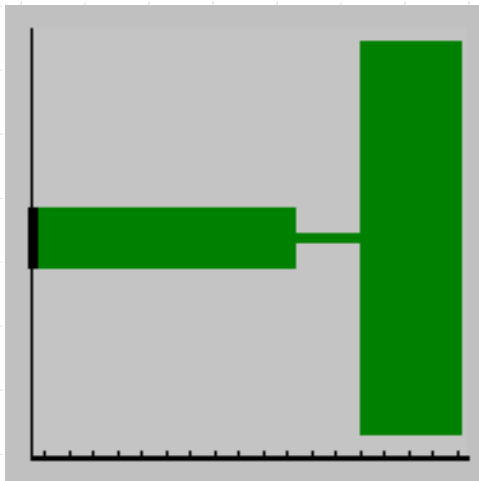
Gain: 7.1dB

Efficiency: 55.3%



Antenna Examples: many others

Edge coupled patch with
matching feed network



Crossed slot coupled patch for dual polarization

