Chapter 7 Mixers

- Three-terminal non-linear or time-varying devices for frequency conversion.
- Implemented by diode and transistor in microwave range.
- Loss, noise and inter-modulation distortion are major parameters.
- Diode mixer and FET mixers will be introduced

7.1 Mixers characteristics

Frequency conversion

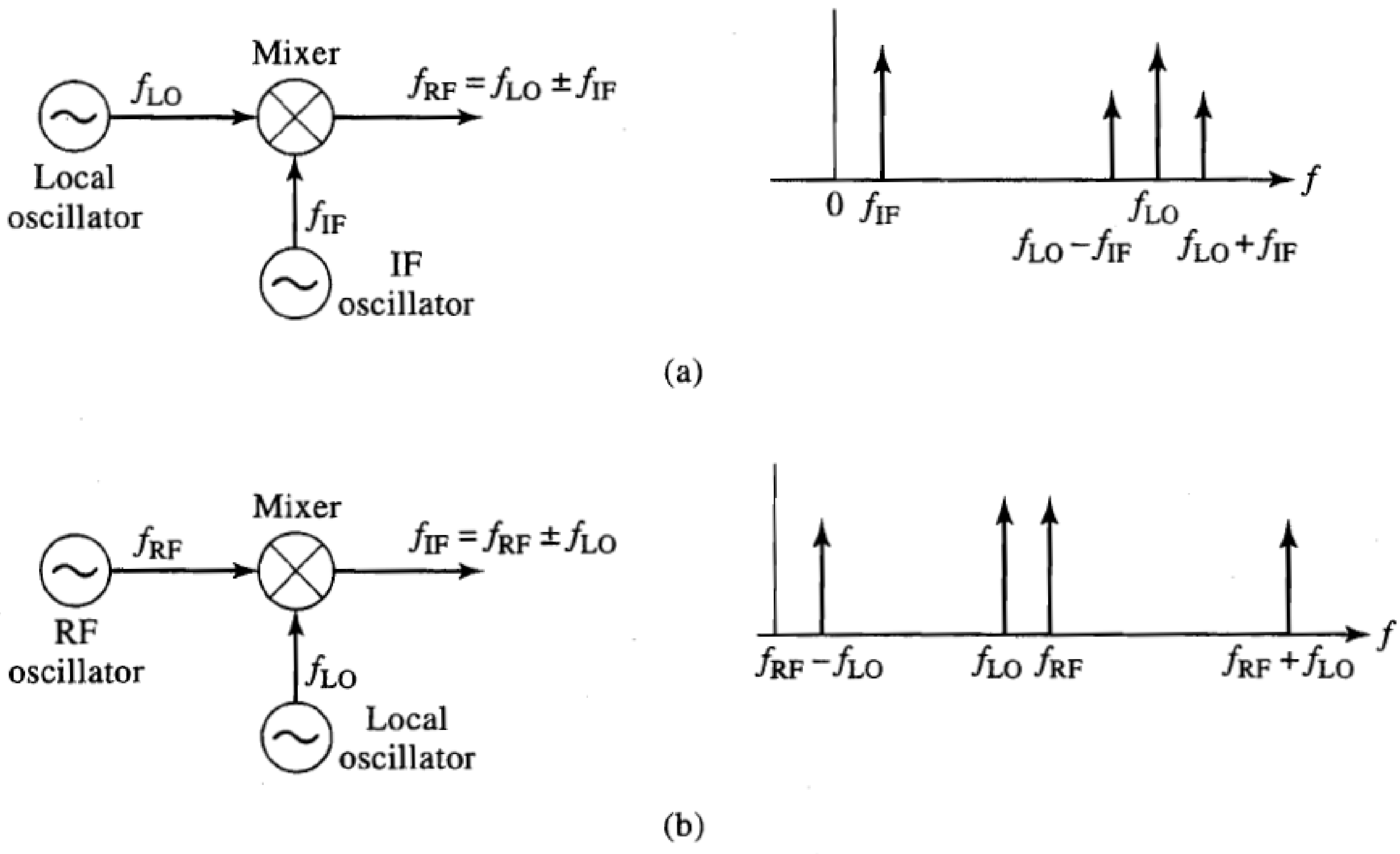
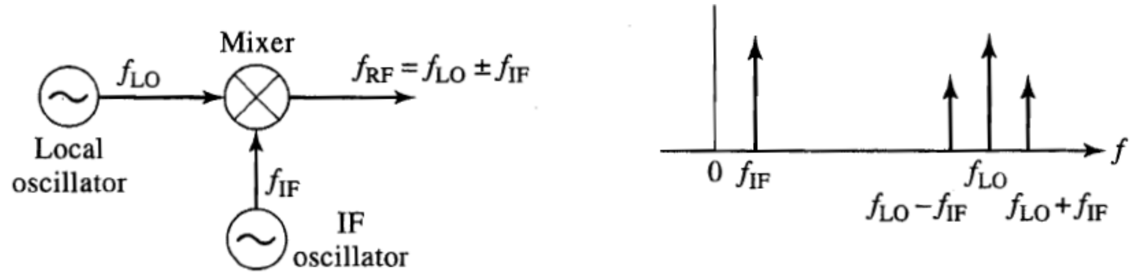


FIGURE 7.1 Frequency conversion using a mixer. (a) Up-conversion. (b) Down-conversion.

- Up-conversion



(a)

$$v_{LO}(t) = \cos 2\pi f_{LO} t. \quad (7.1)$$

$$v_{IF}(t) = \cos 2\pi f_{IF} t. \quad (7.2)$$

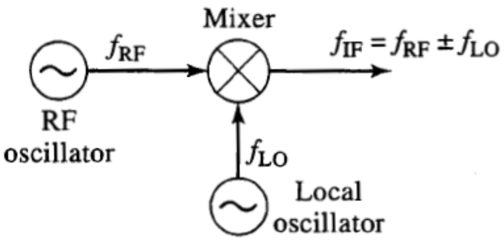
$$v_{RF}(t) = K v_{LO}(t) v_{IF}(t) = K \cos 2\pi f_{LO} t \cos 2\pi f_{IF} t$$
$$= \frac{K}{2}[\cos 2\pi (f_{LO} - f_{IF})t + \cos 2\pi (f_{LO} + f_{IF})t] \quad (7.3)$$

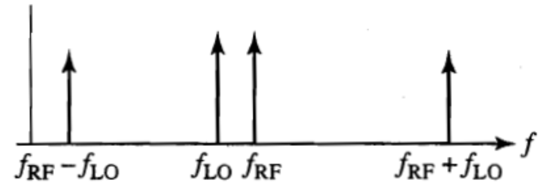
$$f_{RF} = f_{LO} \pm f_{IF} \quad (7.4)$$

$f_{LO} + f_{IF}$ being the upper sideband (USB), and $f_{LO} - f_{IF}$ being the lower sideband (LSB)

A double-sideband (DSB) signal contains both upper and lower sidebands, as in (7.3), while a single-sideband (SSB) signal can be produced by filtering or by using a single-sideband mixer.

- Down-conversion





(b)

$$v_{RF}(t) = \cos 2\pi f_{RF} t, \qquad (7.5)$$

$$v_{LO}(t) = \cos 2\pi f_{LO} t. \qquad (7.1)$$

$$\begin{aligned} v_{IF}(t) &= K v_{RF}(t) v_{LO}(t) = K \cos 2\pi f_{RF} t \cos 2\pi f_{LO} t \\ &= \frac{K}{2} [\cos 2\pi (f_{RF} - f_{LO}) t + \cos 2\pi (f_{RF} + f_{LO}) t] \end{aligned} \qquad (7.6)$$

$$f_{IF} = f_{RF} - f_{LO} \qquad (7.7)$$

Easily selected by low-pass filtering

- Image frequency

$$f_{RF} = f_{LO} + f_{IF}, \tag{7.8a}$$

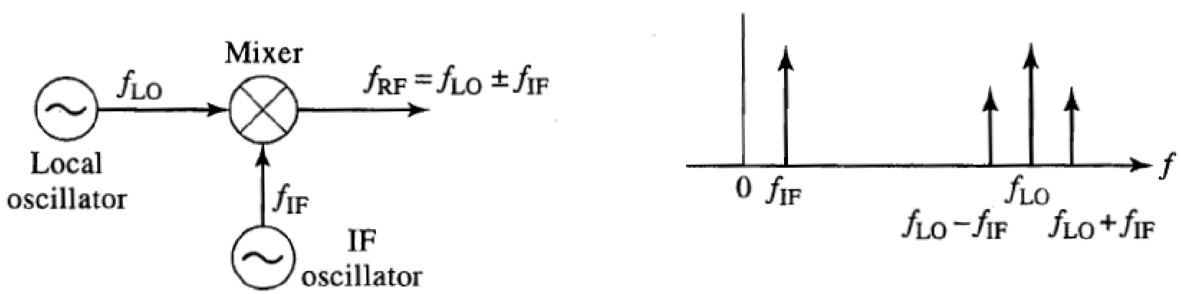
$$f_{IM} = f_{LO} - f_{IF}. \tag{7.8b}$$

The RF frequency defined in (7.8b) is called the image response. The image response is important in receiver design because a received RF signal at the image frequency of (7.8b) is indistinguishable at the IF stage from the desired RF signal of frequency (7.8a), unless steps are taken in the RF stages of the receiver to preselect signals only within the desired RF frequency band.

In practice, most receivers use a local oscillator set at the upper sideband, $f_{LO} = f_{RF} + f_{IF}$, *because this requires a smaller LO tuning ratio when the receiver must* select RF signals over a given band.

- Image frequency

$$f_{LO} = f_{RF} \pm f_{IF}, \tag{7.9}$$



(a)

EXAMPLE 7.1 IMAGE FREQUENCY

The IS-54 digital cellular telephone system uses a receive frequency band of 869–894 MHz, with a first IF frequency of 87 MHz, and a channel bandwidth of 30 kHz. What are the two possible ranges for the LO frequency? If the upper LO frequency range is used, determine the image frequency range. Does the image frequency fall within the receive passband?

Solution

By (7.9), the two possible LO frequency ranges are

$$f_{LO} = f_{RF} \pm f_{IF} = (869 \text{ to } 894) \pm 87 = \begin{cases} 956 \text{ to } 981 \text{ MHz} \\ 782 \text{ to } 807 \text{ MHz} \end{cases}$$

Using the 956–981 MHz LO, (7.7) gives the IF frequency as

$$f_{IF} = f_{RF} - f_{LO} = (869 \text{ to } 894) - (956 \text{ to } 981) = -87 \text{ MHz},$$

so from (7.8b) the RF image frequency range is

$$f_{IM} = f_{LO} - f_{IF} = (956 \text{ to } 981) + 87 = 1043 \text{ to } 1068 \text{ MHz},$$

which is well outside the receive passband. ○

Conversion loss

$$L_c = 10\log \frac{\text{available RF input power}}{\text{available IF output power}} \geq 0 \text{ dB} \tag{7.10}$$

There are inherent losses in the frequency conversion process because of the generation of undesired harmonics and other frequency products.

- 4~7dB for diode mixer at 1~10GHz
- Transistor may even have conversion gain of a few dB.
- Minimum loss for LO power at 0~10dBm.

Noise figure

Noise is generated in mixers by the diode or transistor elements, and by thermal sources due to resistive losses.

NF: 1~5dB typical value.

The noise figure of a mixer depends on whether its input is a single sideband signal or a double sideband signal. This is because the mixer will down-convert noise at both sideband frequencies (since these have the same IF), but the power of a SSB signal is one-half that of a DSB signal (for the same amplitude).

For double side band (DSB) signal

$$v_{\mathrm{DSB}}(t) = A[\cos(\omega_{\mathrm{LO}} - \omega_{\mathrm{IF}})t + \cos(\omega_{\mathrm{LO}} + \omega_{\mathrm{IF}})t]. \tag{7.11}$$

$$v_{\mathrm{IF}}(t) = \frac{AK}{2}\cos(\omega_{\mathrm{IF}}t) + \frac{AK}{2}\cos(-\omega_{\mathrm{IF}}t) = AK\cos\omega_{\mathrm{IF}}t, \tag{7.12}$$

$$S_i = \frac{A^2}{2} + \frac{A^2}{2} = A^2,$$

$$S_o = \frac{A^2K^2}{2}.$$

$$N_o = \frac{(kT_0B + N_{\text{added}})}{L_c}.$$

$$F_{\text{DSB}} = \frac{S_i N_o}{S_o N_i} = \frac{2}{K^2 L_c}\left(1 + \frac{N_{\text{added}}}{kT_0B}\right). \tag{7.13}$$

- **<u>For single side band (SSB) signal</u>**

$$v_{\text{SSB}}(t) = A\cos(\omega_{\text{LO}} - \omega_{\text{IF}})t. \tag{7.14}$$

$$v_{\text{IF}}(t) = \frac{AK}{2}\cos(\omega_{\text{IF}}t). \tag{7.15}$$

$$S_i = \frac{A^2}{2}, \qquad S_o = \frac{A^2K^2}{8}.$$

$$F_{\text{SSB}} = \frac{S_i N_o}{S_o N_i} = \frac{4}{K^2 L_c}\left(1 + \frac{N_{\text{added}}}{kT_0B}\right). \tag{7.16}$$

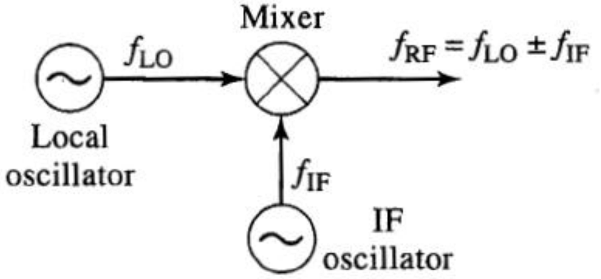
$$F_{\text{SSB}} = 2F_{\text{DSB}}. \tag{7.17}$$

Inter-modulation distortion

Since mixers involve nonlinearity, they will produce intermodulation products.

Typical values of ***P3 for mixers range from 15 dBm to 30 dBm.***

Isolation



Ideally, the LO and **RF ports would be decoupled, but internal impedance mismatches** and limitations of coupler performance often result in some LO power being coupled out of the RF port.

This is a potential problem for receivers that drive the RF port directly from the antenna, because LO power coupled through the mixer to the **RF port will be radiated** by the antenna.

Because such signals will likely interfere with other services or users, the FCC sets stringent limits on the power radiated by receivers.

This problem can be largely alleviated by using a bandpass filter between the antenna and mixer, or by using an RF amplifier ahead of the mixer.

Isolation between the LO and RF ports is highly dependent on the type of coupler used for diplexing these two inputs, but typical values range from 20 **dB to 40 dB.**

7.2 Diode mixers

- Small signal, large signal and ideal switching circuit

Small-signal diode characteristics

$$I(V) = I_s(e^{\alpha V} - 1),$$

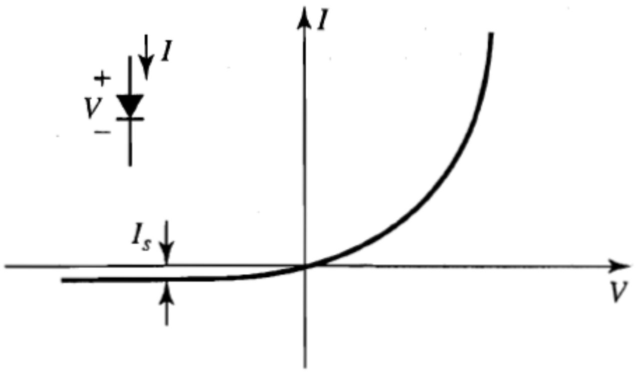


FIGURE 7.2 V-I characteristic of a diode.

where V is the voltage across the diode, I is the current through the diode, and I_s is the reverse saturation current [1]–[3]. The constant $\alpha = q/nkT$, where q is the charge of an electron, k is Boltzmann's constant, T is the temperature in Kelvin, and n is the diode ideality factor. For typical RF diodes, I_s is between 10^{-6} and 10^{-15} A, and α is approximately 1/(28 mV) for $T = 290$ K. The ideality factor, n, depends on the structure of the diode, and can vary from 1.2 for Schottky barrier diodes to about 2.0 for point-contact silicon diodes. The I-V response of (7.18) is shown in Figure 7.2.

$$I = I_0 + i(t), \tag{7.19a}$$

$$V = V_0 + v(t), \tag{7.19b}$$

$$I(V) = I_0 + G_d v(t) + \frac{1}{2} G'_d v^2(t) + \cdots, \tag{7.20}$$

$$G_d = \left. \frac{dI}{dV} \right|_{V=V_0} = \alpha I_s e^{\alpha V_0} = \alpha (I_0 + I_s), \tag{7.21a}$$

$$G'_d = \left. \frac{dG_d}{dV} \right|_{V=V_0} = \left. \frac{d^2 I}{dV^2} \right|_{V=V_0} = \alpha^2 I_s e^{\alpha V_0} = \alpha G_d. \tag{7.21b}$$

- Square-law response term give the frequency conversion.

$$I(V) = I_0 + G_d v(t) + \frac{1}{2} G'_d v^2(t) + \cdots, \tag{7.20}$$

Single-ended mixer

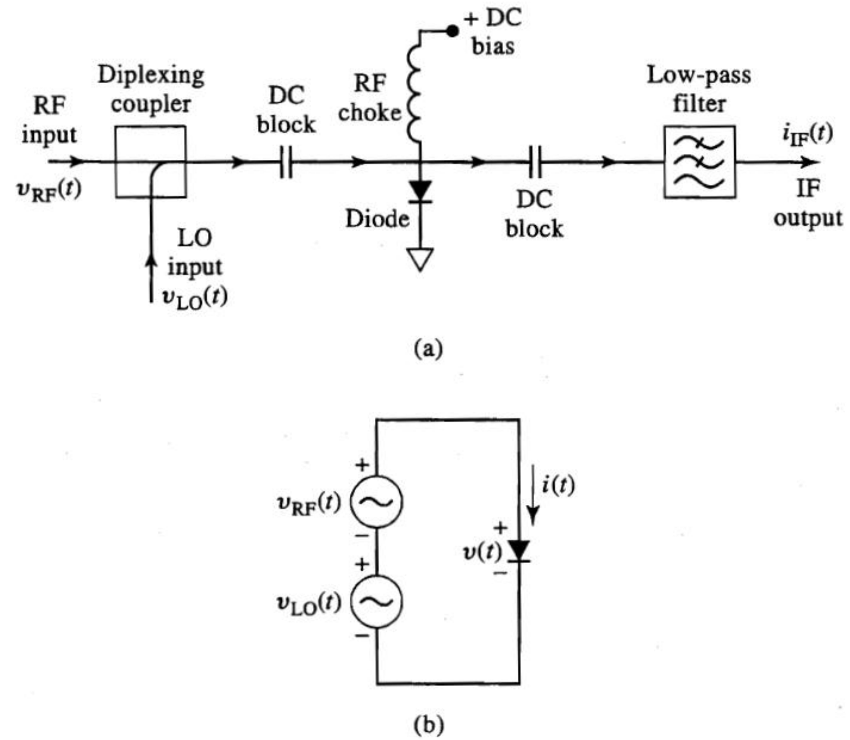


FIGURE 7.3 (a) Circuit for a single-ended mixer. (b) Idealized equivalent circuit of the single-ended mixer.

The RF and LO inputs are combined in a *diplexer, which superimposes the two input voltages to drive the diode. The* diplexing function is easily implemented using an RF coupler or hybrid junction to provide combining as well as isolation between the two inputs.

The diode may be biased with a DC bias voltage, which must be decoupled from the RF signal paths. This is done by using DC blocking capacitors on either side of the diode, and an RF choke between the diode and the bias voltage source.

The AC output of the diode is passed through a low-pass filter to provide the desired IF output voltage.

$$v_{\mathrm{RF}}(t) = V_{\mathrm{RF}} \cos \omega_{\mathrm{RF}} t, \tag{7.22}$$

$$v_{\mathrm{LO}}(t) = V_{\mathrm{LO}} \cos \omega_{\mathrm{LO}} t. \tag{7.23}$$

$$i(t) = I_0 + G_d[v_{\mathrm{RF}}(t) + v_{\mathrm{LO}}(t)] + \boxed{\frac{G'_d}{2}[v_{\mathrm{RF}}(t) + v_{\mathrm{LO}}(t)]^2} + \cdots \tag{7.24}$$

$$\begin{aligned}
i(t) &= \frac{G'_d}{2}[V_{\mathrm{RF}} \cos \omega_{\mathrm{RF}} t + V_{\mathrm{LO}} \cos \omega_{\mathrm{LO}} t]^2 \\
&= \frac{G'_d}{2}\left[V_{\mathrm{RF}}^2 \cos^2 \omega_{\mathrm{RF}} t + 2V_{\mathrm{RF}} V_{\mathrm{LO}} \cos \omega_{\mathrm{RF}} t \cos \omega_{\mathrm{LO}} t + V_{\mathrm{LO}}^2 \cos^2 \omega_{\mathrm{LO}} t\right] \\
&= \frac{G'_d}{4}\left[V_{\mathrm{RF}}^2(1 + \cos 2\omega_{\mathrm{RF}} t) + V_{\mathrm{LO}}^2(1 + \cos 2\omega_{\mathrm{LO}} t)\right. \\
&\quad \left. + \boxed{2V_{\mathrm{RF}} V_{\mathrm{LO}} \cos(\omega_{\mathrm{RF}} - \omega_{\mathrm{LO}})t} + 2V_{\mathrm{RF}} V_{\mathrm{LO}} \cos(\omega_{\mathrm{RF}} + \omega_{\mathrm{LO}})t\right]
\end{aligned}$$

$$i_{\mathrm{IF}}(t) = \frac{G'_d}{2} V_{\mathrm{RF}} V_{\mathrm{LO}} \cos \omega_{\mathrm{IF}} t, \tag{7.25}$$

Large-signal

Small-signal analysis of a mixer can not accurate enough to provide a realistic result for conversion loss.

This is primarily because the power supplied to the mixer LO port is usually large enough to violate the small-signal approximation.

Here, a fully nonlinear analysis of a resistive diode mixer. Reactance associated with the diode junction and package are ignored, to simplify the analysis.

Low level RF input voltage, and a much larger LO pump signal.

These two AC input signals generate a multitude of harmonics and other frequency products:

ω_{RF}	RF input signal (low power)
$\omega_{IF} = \omega_{RF} - \omega_{LO}$	IF output signal (low power)
$\omega_{IM} = \omega_{LO} - \omega_{IF}$	image signal (low power)
ω_{LO}	LO input signal (high power)
$n\omega_{LO}$	harmonics of LO (high power)
$n\omega_{LO} \pm \omega_{IF}$	harmonic sidebands of LO (low power)

Large Signal

In a typical mixer, harmonics of the LO and the harmonic sidebands are terminated reactively, and therefore do not lead to much power loss.

This leaves three signal frequencies of most importance: ω_{RF}, ω_{IF}, and ω_{IM}.

$$\omega_{\mathrm{IM}} = 2\omega_{\mathrm{LO}} - \omega_{\mathrm{RF}}$$

Under the assumption that the RF input voltage is small, we can write the AC diode current as a Taylor series expansion about the LO voltage as

$$i(v) = I(v_{\mathrm{LO}}) + v\frac{dI}{dV}\bigg|_{v_{\mathrm{LO}}} + \frac{1}{2}v^2\frac{d^2I}{dV^2}\bigg|_{v_{\mathrm{LO}}} + \cdots \tag{7.26}$$

The expansion point here is about the LO voltage.

The <u>second term</u> is a function of the RF and LO input voltages, and will provide a good approximation for the three products at frequencies ω_{RF}, ω_{IF}, and ω_{IM}.

, with a large LO pump signal.

$$g(t) = \frac{dI}{dV}\bigg|_{v_{\mathrm{LO}}} = \alpha I_s e^{\alpha V}\Big|_{v_{\mathrm{LO}}} = \alpha I_s e^{\alpha V_{\mathrm{LO}} \cos \omega_{\mathrm{LO}} t}.$$

$$i(t) = g(t)v(t)$$

$$g(t) = g_0 + \sum_{n=1}^{\infty} 2g_n \cos n\omega_{\mathrm{LO}} t,$$

$$g_n = \frac{1}{T}\int_0^T g(t)\cos n\omega_{\mathrm{LO}} t\, dt = \frac{\alpha\omega_{\mathrm{LO}} I_s}{2\pi}\int_0^{\frac{\omega_{\mathrm{LO}}}{2\pi}} e^{\alpha V_{\mathrm{LO}}\cos\omega_{\mathrm{LO}} t}\cos n\omega_{\mathrm{LO}} t\, dt$$

$$= \frac{\alpha I_s}{2\pi}\int_0^{2\pi} e^{\alpha V_{\mathrm{LO}}\cos\theta}\cos n\theta\, d\theta = \alpha I_s I_n(\alpha V_{\mathrm{LO}})$$

$$i(t) = I_{\mathrm{RF}}\cos\omega_{\mathrm{RF}} t + I_{\mathrm{IF}}\cos\omega_{\mathrm{IF}} t + I_{\mathrm{IM}}\cos\omega_{\mathrm{IM}} t,$$

Large Signal

$$i(t) = g(t)v(t)$$

$$g(t) = g_0 + \sum_{n=1}^{\infty} 2g_n \cos n\omega_{\mathrm{LO}}t,$$

$$g_n = \frac{1}{T}\int_0^T g(t)\cos n\omega_{\mathrm{LO}}t\,dt = \frac{\alpha\omega_{\mathrm{LO}}I_s}{2\pi}\int_0^{\frac{\omega_{\mathrm{LO}}}{2\pi}} e^{\alpha V_{\mathrm{LO}}\cos\omega_{\mathrm{LO}}t}\cos n\omega_{\mathrm{LO}}t\,dt$$

$$= \frac{\alpha I_s}{2\pi}\int_0^{2\pi} e^{\alpha V_{\mathrm{LO}}\cos\theta}\cos n\theta\,d\theta = \alpha I_s I_n(\alpha V_{\mathrm{LO}})$$

AC diode current consist of three components at the frequencies ω_{RF}, ω_{IF}, and ω_{IM}:

$$i(t) = I_{RF}\cos\omega_{RF}t + I_{IF}\cos\omega_{IF}t + I_{IM}\cos\omega_{IM}t, \tag{7.31}$$

where I_{RF}, I_{IF}, and I_{IM} are the amplitudes of the RF, IF, and image signals to be determined. If the RF voltage of (7.22) is applied to the diode through a source resistance R_g, and the IF and image ports are terminated in load resistances R_{IF} and R_g, respectively, then the voltage

$$v(t) = V_{RF}\cos\omega_{RF}t - I_{RF}R_g\cos\omega_{RF}t - I_{IF}R_{IF}\cos\omega_{IF}t - I_{IM}R_g\cos\omega_{IM}t.$$

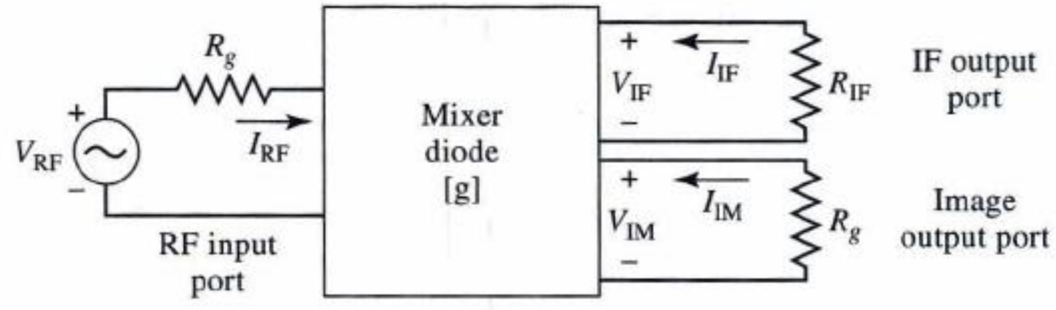


FIGURE 7.4 Equivalent circuit for the large-signal model of the resistive diode mixer.

$$g(t) = g_0 + 2g_1 \cos \omega_{\mathrm{LO}} t + 2g_2 \cos 2\omega_{\mathrm{LO}} t.$$

$$\begin{bmatrix} I_{\mathrm{RF}} \\ I_{\mathrm{IF}} \\ I_{\mathrm{IM}} \end{bmatrix} = \begin{bmatrix} g_0 & g_1 & g_2 \\ g_1 & g_0 & g_1 \\ g_2 & g_1 & g_0 \end{bmatrix} \begin{bmatrix} V_{\mathrm{RF}} - I_{\mathrm{RF}} R_g \\ -I_{\mathrm{IF}} R_{\mathrm{IF}} \\ -I_{\mathrm{IM}} R_g \end{bmatrix}, \tag{7.34}$$

where V_{RF} is the source voltage, and the g_n's are defined in (7.30). Note that multiplication of (7.32) by (7.33) creates several frequencies in addition to ω_{RF}, ω_{IF}, and ω_{IM}, but we assume these frequencies to be reactively terminated so that they do not lead to significant power dissipation.

The easiest way to find the available power from the IF port is to first find the Norton equivalent source for the IF port.

$$I_{SC} = -I_{IF}|_{R_{IF}=0} = \frac{g_1 V_{RF}}{1 + g_0 R_g + g_2 R_g}.$$

The open-circuit IF port voltage is found by setting $I_{IF} = 0$, and solving (7.34) for V_{IF}:

$$V_{OC} = V_{IF}|_{I_{IF}=0} = \frac{g_1 V_{RF}}{2g_1^2 R_g - g_0 g_2 R_g - g_0(1 + g_0 R_g)}. \tag{7.36}$$

Then the Norton conductance of the IF port is

$$G_{IF} = \frac{I_{SC}}{V_{OC}} = g_0 - \frac{2g_1^2 R_g}{1 + g_0 R_g + g_2 R_g}.$$

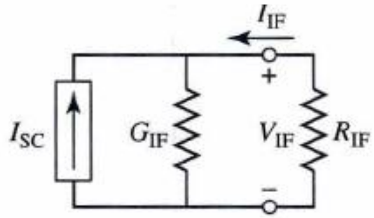


FIGURE 7.5 Norton equivalent circuit for the IF port of the large-signal model of the resistive diode mixer.

The available output power at the IF port is

$$P_{\mathrm{IF-avail}} = \frac{|I_{\mathrm{SC}}|^2}{4G_{\mathrm{IF}}},$$

The available input power from the **RF source is**

$$P_{\mathrm{RF-avail}} = \frac{|V_{\mathrm{RF}}|^2}{4R_g}.$$

So from (7.10) the conversion loss is (not in dB)

$$L_c = \frac{P_{\mathrm{RF-avail}}}{P_{\mathrm{IF-avail}}} = \frac{(1 + g_0 R_g + g_2 R_g)\left[g_0(1 + g_0 R_g + g_2 R_g) - 2g_1^2 R_g\right]}{g_1^2 R_g}. \tag{7.39}$$

Note that the conversion loss does not depend on the IF port termination, R_{IF}, because of the use of available powers. It does depend on R_g, the RF and image port terminations, so

it is possible to minimize the conversion loss by properly selecting R_g. If we let $x = 1/R_g$, $a = g_0 + g_2$, and $b = 2g_1^2/g_0$, then (7.39) can be rewritten as

$$L_c = \frac{2(x + a)(x + a - b)}{bx}. \tag{7.40}$$

Differentiating with respect to x and setting the result to zero gives the optimum value of x as

$$x_{\text{opt}} = \sqrt{a(a-b)}, \tag{7.41}$$

for which the minimum value of conversion loss is

$$\begin{aligned} L_{c-\min} &= \frac{2[a+\sqrt{a(a-b)}][a-b+\sqrt{a(a-b)}]}{b\sqrt{a(a-b)}} \\ &= \frac{2[\sqrt{a}+\sqrt{a-b}]^2}{b} = 2\frac{1+\sqrt{1-b/a}}{1-\sqrt{1-b/a}}. \end{aligned} \tag{7.42}$$

We can evaluate this result by approximating values for g_0, g_1, and g_2. For an LO input power of 10 mW, V_{LO} is about 0.707 V rms, and $\alpha = 1/28$ mV, so αV_{LO}, the argument of the modified Bessel functions for g_n given in (7.30), is approximately 25. Thus the modified Bessel functions can be approximated asymptotically using the large-argument formula given in Appendix B, and the g_n's simplified as

$$g_n = \alpha I_s I_n(\alpha V_{\text{LO}}) \cong \alpha I_s \frac{e^{\alpha V_{\text{LO}}}}{\sqrt{2\pi\alpha V_{\text{LO}}}}. \tag{7.43}$$

Thus

$$\frac{b}{a} = \frac{2g_1^2}{g_0(g_0+g_2)} \cong 1, \tag{7.44}$$

The minimum conversion loss of (7.42) reduces to L_C *= 2, or 3 dB. This means that* halt the RF input power is converted to IF power, and half is converted to power at the image frequency.